

Name:	Level/Subject: 4049 Sec 4 A-Math
Material: June Practice Questions 2022	Centre: Overmugged

Instructions

- Answer all questions
- If working is needed for any question it must be shown with the answer
- Omission of essential working will result in loss of marks
- You are expected to use a scientific calculator to evaluate explicit numerical expressions
- If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures
- Give answers in degrees to one decimal place
- For π , use either your calculator value of 3.142, unless the question requires the answer in terms of π
- A copy of the formula list is provided for you on the next page

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Question Source

All questions are sourced and selected based on the known abilities of students sitting for the 'O' Level A-Math Examination. All questions compiled here are from **2009 - 2021 School Mid-Year / Prelim Papers**. Questions are categorised into the various topics and range in varying difficulties. If questions are sourced from respective sources, credit will be given when appropriate.

How to read:

[S4 ABCSS P1/2011 PRELIM Qn 1]

Secondary 4, ABC Secondary School, Paper 1, 2011, Prelim, Question 1

Prepared by: **Kaiwen** :)

This question paper consists of 36 printed pages including the cover page

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List of Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Expansion

$$(a + b)^n = a^n + \binom{n}{1} a^{n-1}b + \binom{n}{2} a^{n-2}b^2 + \dots + \binom{n}{r} a^{n-r}b^r + \dots + b^n$$

where n is a positive integer and

$$\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\dots(n-r+1)}{r!}$$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for $\triangle ABC$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2}bc \sin A$$

1 Questions

1.1 Quadratic Equations & Inequalities

1. (a) Given the following curve

$$y = 4x^2 + px + p - 6$$

find the possible range of value(s) of p for which

- (i) the curve intersects the line $y = -3$ [3]
 - (ii) the line $y = -3$ is a tangent to the curve [1]
 - (iii) the curve has a positive y -intercept [1]
- (b) Show that the following has real and distinct roots for all real values of m [3]

$$(m+1)x^2 + (4m+3)x + 2m-1 = 0$$

Credit: **S4 FMS(S) P1/2015 PRELIM Qn 8**

2. (a) (i) Show that the expression, where p is a constant, is always positive for all real values of x [3]

$$x^2 + px - x + p^2 + 2$$

- (ii) **Hence**, find the range of values of x for which [4]

$$\frac{x^2 - 3x - 28}{x^2 + px - x + p^2 + 2} < 0$$

- (b) Find the range of values of k for which the line cuts the curve at 2 distinct points [4]

$$y = 2x - k$$

$$y^2 = x + k$$

Credit: **S4 TSS P2/2015 PRELIM Qn 2 (MODIFIED)**

3. (a) Find the range of values of x for which [3]

$$3(2x-5)^2 > x(2x-5)$$

- (b) The equation of the curve, where k is a constant is given as such

$$y = (k+4)x^2 + 4x - k$$

- (i) Show that the curve meets the x -axis for all possible values of k [3]
 - (ii) Find the value of k for which the x -axis is a tangent to the curve [1]
- (c) Given that the following equation is always positive, what conditions must be applied to the constants p and q ? [2]

$$y = px^2 + 4x + q$$

Credit: **S4 XMSS P1/2016 PRELIM Qn 7**

4. (a) Show that the roots of the equation are real for all values of a [3]

$$x^2 + (a-2)x = 2a$$

- (b) Show that there are no values of b for which the curve is always positive [4]

$$y = (b-3)x^2 - 2bx + (b-2)$$

Credit: **S4 GMS(S) P2/2016 PRELIM Qn 4**

1.2 (Indices) and Surds

1. The area of a triangle is $\left(1 + \frac{5\sqrt{5}}{2}\right) \text{ cm}^2$. If the length of the base of the triangle is $(3 + 2\sqrt{5}) \text{ cm}$, [4]
find, without using a calculator, the height of the triangle in the form of $(a + b\sqrt{5}) \text{ cm}$, where a and b are integers

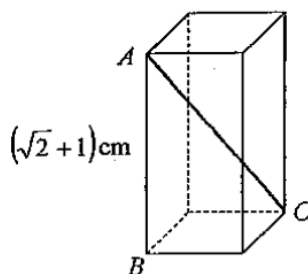
Credit: **S4 CCHS(M) P1/2016 PRELIM Qn 1**

2. (a) Find the value of k and n given that [3]

$$\frac{2(4)^{\frac{1}{2}x+2} - 2^{x+1}}{6^x \times 3^{1-2x}} = k(3)^{nx}$$

Credit: **S4 TSS P1/2015 PRELIM Qn 3(a)**

- (b) The diagram below shows a cuboid with a square base



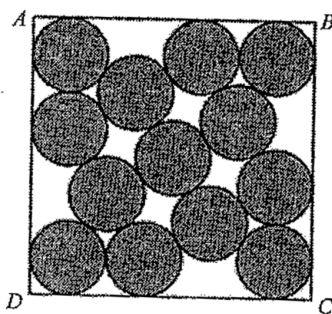
The height AB of the cuboid is $(\sqrt{2} + 1) \text{ cm}$ and the length of the diagonal AC is $\frac{7\sqrt{2}}{2\sqrt{2} + 1} \text{ cm}$

- (i) Express the length of the diagonal AC in the form $a + b\sqrt{2}$, where a and b are integers [2]
(ii) Find an expression of BC^2 in the form $c + d\sqrt{2}$, where c and d are integers [4]
(iii) Express the volume of the cuboid in the following form, where k is an integer [3]

$$V = \frac{5}{2}(\sqrt{2} + k) \text{ cm}^3$$

Credit: **S4 CCHS(Y) P2/2016 PRELIM Qn 4**

3. The diagram shows a maximum number of 13 identical circles packed into a square



If the radius of each circle is 1 cm

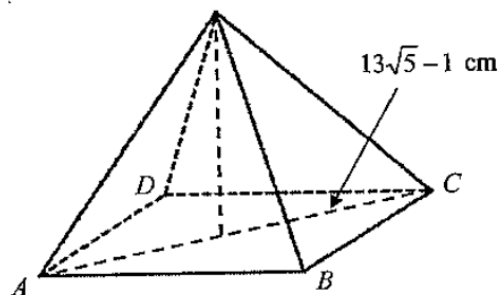
- (a) find the exact length of AD [3]
(b) express the area of the square in the form $(a + b\sqrt{3}) \text{ cm}^2$ [2]

Credit: **S4 CWSS P1/2016 PRELIM Qn 7**

4. (a) Express the following in the form $a + b\sqrt{5}$, where a and b are integers [2]

$$\frac{6\sqrt{5}}{2\sqrt{5} - 4}$$

- (b) The diagram shows a pyramid with a square base $ABCD$

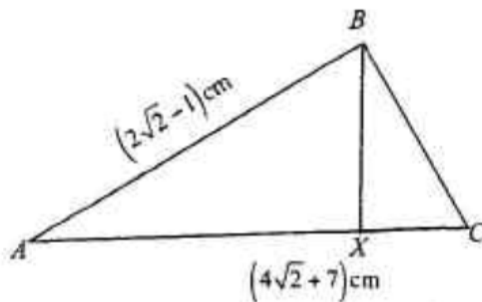


The diagonal AC of the square base is $(13\sqrt{5} - 1)$ cm and the height of the pyramid is $\frac{6\sqrt{5}}{2\sqrt{5} - 4}$ cm

- (i) find an expression for AC^2 in the form $c + d\sqrt{5}$, where c and d are integers [2]
 (ii) express the volume of the pyramid in the form $(m + n\sqrt{5}) \text{ cm}^3$, where m and n are integers [4]

Credit: S4 FHSS P2/2016 PRELIM Qn 5

5. The diagram shows a triangle ABC such that $AB = (2\sqrt{2} - 1)$ cm and $AC = (4\sqrt{2} + 7)$ cm. The point X lies on AC such that $\angle AXB = \angle ABC$



- (a) Show that [2]

$$AX \times AC = AB^2$$

- (b) Find an expression for AX in the form [4]

$$\frac{1}{17} (a + b\sqrt{2})$$

- (c) Show that $\angle AXB = 90^\circ$ given that [3]

$$BC^2 = 72 + 60\sqrt{2}$$

Credit: S4 MSHS P1/2016 PRELIM Qn 5

1.3 Polynomials

1. Given that

$$f(x) = 6x^3 + 3x^2 - x + 2$$

- (a) show that the equation $f(x) = 0$ has only one real root. Find the value of the real root [5]
(b) sketch the curve, showing clearly the x and y intercepts [2]

Credit: **S4 BHSS P1/2016 PRELIM Qn 5**

2. Given the cubic expression $f(x)$ has a factor of $(x + 2)$ and leaves a remainder of 6 when divided by $(x + 1)$

$$f(x) = x^3 + px^2 + qx + 4$$

- (a) find the value of p and of q [4]
(b) factorise $f(x)$ completely [2]

Credit: **S4 MGS P1/2016 PRELIM Qn 5**

3. (a) Find the value of m , where $m > 0$, for which $2x^2 + x + m$ is a factor of $4x^3 + 5x - 3$ [3]
(b) The cubic polynomial $f(x)$ is such that the coefficient of x^3 is 3 and the roots of the equation $f(x) = 0$ are -2 , 3 and k . Given that $f(x)$ has a remainder of 42 when divided by $(x + 1)$, find
(i) the value of k [3]
(ii) the remainder when $f(x)$ is divided by x [2]

Credit: **S4 SCGS P2/2016 PRELIM Qn 4**

4. The function $f(x)$, where A is a constant, leaves a remainder of $1\frac{3}{8}$ when divided by $(2x - 1)$

$$f(x) = 1 + 2x + Ax^2 - x^3$$

- (a) Find the value of A [2]
(b) **Hence**, solve the equation $f(x) = 0$, giving your answers in the exact form [4]

Credit: **S4 TKGS P2/2016 PRELIM Qn 2**

5. Given that $4x^2 + 7x - 2$ is a factor of

$$f(x) = a(x^4 + 1) + 7x^3 - 10x^2 + bx$$

- (a) Show that $a = 4$ and $b = -14$ [5]
(b) Find the remainder when $f(x)$ is divided by $(x + 1)$ [2]

Credit: **S4 VS P1/2016 PRELIM Qn 4**

1.4 Partial Fractions

1. (a) Express the following as the sum of 3 partial fractions [4]

$$\frac{8x - 5}{x^2(1 - x)}$$

- (b) **Hence**, find [2]

$$\int \frac{8x - 5}{x^2(1 - x)} dx$$

Credit: **S4 CCHS(Y) P1/2015 PRELIM Qn 9**

2. (a) Express the following in partial fractions [5]

$$\frac{3x^2 + 10x}{(x + 2)(x^2 - 4)}$$

- (b) **Hence**,

- (i) Find [2]

$$\int \frac{3x^2 + 10x}{(x + 2)(x^2 - 4)} dx$$

- (ii) Show that [2]

$$\int_3^4 \frac{3x^2 + 10x}{(x + 2)(x^2 - 4)} dx = \ln\left(\frac{24}{5}\right) + \frac{1}{15}$$

Credit: **S4 VS P2/2015 PRELIM Qn 5**

3. (a) Express the following in partial fractions [3]

$$\frac{11 - 7x}{3x^2 + 11x - 4}$$

- (b) **Hence**, evaluate [4]

$$\int_1^2 \frac{11 - 7x}{9x^2 + 33x - 12} dx$$

Credit: **S4 HIHS P2/2016 PRELIM Qn 4**

4. (a) Given that

$$f(x) = 3x^3 + 11x^2 + 8x - 4$$

- (i) Show that $3x - 1$ is a factor of $f(x)$ [1]

- (ii) **Hence**, factorise the cubic polynomial $f(x)$ completely [2]

- (b) Express the following as the sum of 3 partial fractions [4]

$$\frac{5x^2 - 2x + 11}{3x^3 + 11x^2 + 8x - 4}$$

- (c) **Hence**, find [3]

$$\int \frac{5x^2 - 2x + 11}{3x^3 + 11x^2 + 8x - 4} dx$$

Credit: **S4 MSHS P2/2016 PRELIM Qn 8**

1.5 Binomial Theorem

1. When $(1 - 2p)^n$ is expanded in ascending powers of p , the sum of the constant term, coefficients of p and p^2 is 161. If n is a positive integer, find the value of n [4]

Credit: **S4 ANDSS P2/2015 PRELIM Qn 1**

2. (a) (i) Write down, and simplify, the first four terms in the expansion of the following in ascending powers of x [2]

$$(1 - x)^8$$

- (ii) Determine the coefficient of z^3 in the expansion of [3]

$$(1 - 2z - z^2)^8$$

- (b) (i) Write down the general term in the binomial expansion of [1]

$$\left(2x - \frac{1}{3x^3}\right)^6$$

- (ii) Determine if there is a constant term in the expansion [1]

- (iii) Using the general term, or otherwise, determine the coefficient of x^2 in the binomial expansion [2]

$$\left(3x^4 + 2 - \frac{3}{x}\right)\left(2x - \frac{1}{3x^3}\right)^6$$

Credit: **S4 ANDSS P2/2016 PRELIM Qn 4**

3. (a) Find the term independent of x in the binomial expansion of [3]

$$\left(x^2 - \frac{1}{2x^3}\right)^{10}$$

- (b) Given that the first 4 terms in the binomial expansion of the following, where a and b are constants

$$\left(2x + \frac{1}{4}\right)^9 = 512x^9 + 576x^8 + ax^7 + bx^6 + \dots$$

find

- (i) the value of a and of b [3]

- (ii) the coefficient of x^6 in [2]

$$\left(2x + \frac{1}{4}\right)^9 \left(\frac{4}{x} - 1\right) \left(\frac{4}{x} + 1\right)$$

Credit: **S4 CGSS P2/2016 PRELIM Qn 3**

4. (a) Write down and simplify the first 3 terms of the expansion, in ascending powers of x of
- (i) $(1 + 6x)^6$ [1]
- (ii) $(1 - kx)^6$ [1]
- (b) Using your results from part (a), obtain the coefficient of x^2 , in terms of k , in the expansion of
- $$[1 + (6 - k)x - 6kx^2]^6$$
- (c) Using your answer in part (b), where k is an integer, the coefficient of x^2 is 168. Find k [2]

Credit: **S4 CHIJ Sec P1/2016 PRELIM Qn 1**

5. (a) Given that n is a positive integer, write down, without simplifying, the $(r+1)$ th term in the binomial expansion of
- $$\left(\frac{x}{2} - \frac{k}{x^2}\right)^n$$
- (b) The binomial expansion in part (a) has a constant term. Show that n is a multiple of 3 [1]
- (c) Given that $n = 9$ and the constant term is [3]

$$-\frac{2625}{2}$$

find the value of k

- (d) Using the value of k found in part (c), find the term independent of x in the expansion of [3]

$$(2 + x^3) \left(\frac{x}{2} - \frac{k}{x^2}\right)^9$$

Credit: **S4 MSHS P1/2016 PRELIM Qn 4**

1.6 Exponential & Logarithms

1. (a) Solve the equation [4]

$$\log_3 4 - \log_9 (x^2 + 4x + 4) = \log_{\frac{1}{3}} x$$

- (b) (i) Sketch the graph of [1]

$$y = e^{-x}$$

- (ii) In order to solve the following equation, a graph of a suitable straight line is drawn on the same set of axes as the graph of $y = e^{-x}$. Find the equation of the straight line [2]

$$\ln \left(\frac{1}{\sqrt{x-3}} \right) = \frac{1}{2}x$$

Credit: **S4 CCHS(Y) P2/2015 PRELIM Qn 1**

2. (a) Find the values of a and b such that, for all positive values of y [3]

$$\lg \left(\frac{125}{y} \right) = a \lg(by) - 4 \lg y$$

- (b) Solve the equation [5]

$$2 \log_5 e^x + \frac{1}{\log_2 5} = \log_5 (2 - 3e^x)$$

Credit: **S4 TSS P1/2015 PRELIM Qn 3(b) & (c)**

3. In January 2016, Adam bought an antique vase for \$1500. It was believed that the value of the antique vase will increase continuously with time such that it doubles after every 5 years [2]
- (a) Formulate an expression for $\$V$, the value of the vase after Adam owned it for x years [2]
- (b) Sketch the graph of V against x [2]
- (c) Using your answer in part (a), find the number of years that Adam has to wait before the value of the vase appreciates to one million dollars [3]

Credit: **S4 ANDSS P1/2016 PRELIM Qn 8**

4. (a) Solve for y in [3]

$$\log_a 2y^2 + \log_a 8 + \log_a 16y - \log_a 64y = 2 \log_a 4$$

- (b) If $x = \lg m$ is a solution of the equation [3]

$$10^{2x+1} + 7(10^x) = 26$$

Find the value of m

Credit: **S4 CCHS(Y) P1/2016 PRELIM Qn 7**

5. The number of people, N , in a housing estate who contracted influenza during a flu epidemic after t days is modelled by the equation

$$N = \frac{1000}{1 + 199e^{-0.8t}}$$

- (a) Find the initial number of people who contracted influenza during the flu epidemic [1]
- (b) Given that there are 937 people who contracted influenza after x days, find x correct to the nearest whole number [3]
- (c) Find the number of people who eventually contracted influenza after a long time [1]

Credit: **S4 CGSS P1/2016 PRELIM Qn 5**

1.7 Trigonometry

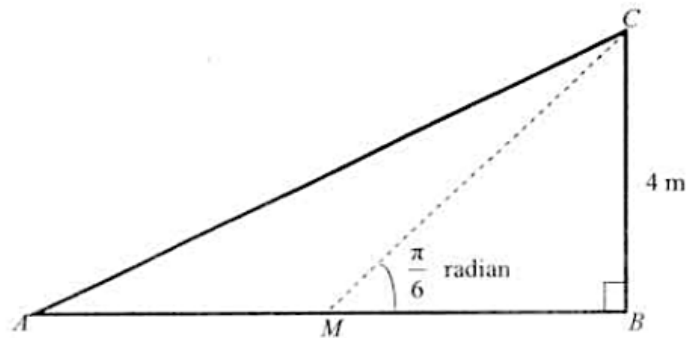
1. (a) (i) Solve the equation, for $0^\circ \leq y \leq 360^\circ$ [3]

$$\sin^2 y + 2 \cos 2y = 2 \cos y$$

- (ii) Prove that [4]

$$\frac{\cos(A+B) + \cos(A-B)}{\sin(A+B) - \sin(A-B)} = \cot B$$

- (b) The diagram shows a triangle ABC in which $\angle CMB$ is $\frac{\pi}{6}$ radians, $\angle B$ is a right angle, M is the midpoint of AB and the length of CB is 4 m [6]



Without using a calculator, find the value of the integer k such that

$$\angle ACM = \sin^{-1} \left(\frac{\sqrt{k}}{26} \right)$$

Credit: S4 CHS P2/2015 PRELIM Qn 2 & 3

2. (a) On the **same axes**, sketch the graphs of the following, $0 \leq t \leq 4\pi$ [4]

$$y = 2 \sin t + 2 \qquad y = \frac{1}{2} \sin \frac{t}{2} + 2$$

- (b) It is observed that the height, y m, above the sea-level, reached by ocean waves on two particular days during a time interval of 4π minutes can be modelled by trigonometric functions. The function $y = 2 \sin t + 2$ models the height of waves on Day 1, and the function $y = \frac{1}{2} \sin \frac{t}{2} + 2$ models the height of waves on Day 2. With reference to the graphs that you have sketched in part (i),
- (i) state the number of instances when the waves on the 2 days reached the same height during the time interval $0 < t < 4\pi$. Justify your answer [2]
- (ii) Which of the 2 days would have provided surfers with a more thrilling experience of riding the waves at sea? Explain your answer [3]

Credit: S4 PLMGS P2/2015 PRELIM Qn 9

3. (a) Prove that [4]

$$\frac{2 \cos 2A + \cos A + 2}{2 \sin 2A + \sin A} = \cot A$$

- (b) **Hence**, solve the equation, for $0 \leq x \leq \pi$ [4]

$$\frac{2 \cos 6x + \cos 3x + 2}{2 \sin 6x + \sin 3x} = 5$$

Credit: S4 TKSS P2/2015 PRELIM Qn 4

4. (a) (i) Show that

[2]

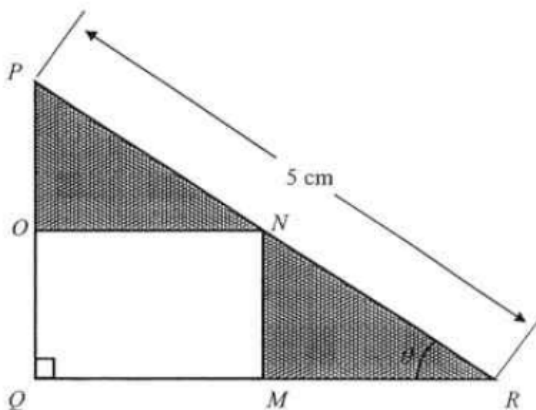
$$\frac{(\cos \theta + \sin \theta)^2}{\sec^2 \theta + 2 \tan \theta} = \cos^2 \theta$$

- (ii)
- Hence**
- , find all values of
- θ
- , where
- $0 < \theta < 2\pi$
- , which satisfy the equation

[4]

$$\frac{\sec^2 \theta + 2 \tan \theta}{(\cos \theta + \sin \theta)^2} = 2(2 + \tan \theta)$$

- (b) The diagram shows a rectangle
- $MNOQ$
- embedded in a triangle
- PQR
- . It is given that
- $PR = 5$
- cm,
- $\angle QRP = \theta$
- and area of rectangle
- $MNOQ$
- is
- $\frac{25}{4} \cos^2 \theta$
- where
- $0^\circ < \theta < 90^\circ$



- (i) Show that the shaded area,
- A
- , is given by

[3]

$$A = \frac{25}{8}(2 \sin 2\theta - \cos 2\theta) - \frac{25}{8}$$

- (ii)
- Hence**
- , show that
- A
- can be expressed in the form

[2]

$$A = R \sin(2\theta - \alpha) - \frac{25}{8} \quad R > 0, 0^\circ < \alpha < 90^\circ$$

- (iii) State the exact maximum value of
- A

[1]

Credit: S4 NGHS P2/2015 PRELIM Qn 2 & 3

5. (a) Prove that

[3]

$$\frac{1 - \tan^2 \theta}{1 + \tan^2 \theta} = \cos 2\theta$$

- (b) Using the result in part (a) to show that, if we let
- $x = \tan 67.5^\circ$

[2]

$$1 + x^2 = \sqrt{2}x^2 - \sqrt{2}$$

- (c)
- Hence**
- , find the value of the constants
- c
- and
- d
- such that

[3]

$$\tan 67.5^\circ = c + d\sqrt{2}$$

- (d)
- Hence**
- , show that

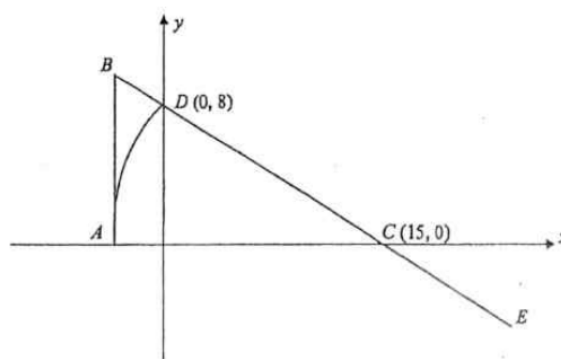
[3]

$$\tan 7.5^\circ = \frac{1 + \sqrt{2} - \sqrt{3}}{1 + \sqrt{3} + \sqrt{6}}$$

Credit: S4 CGSS P2/2016 PRELIM Qn 8

1.8 Coordinate Geometry

1. In the diagram, ADC is a sector of a circle with centre C and $BDCE$ is a straight line



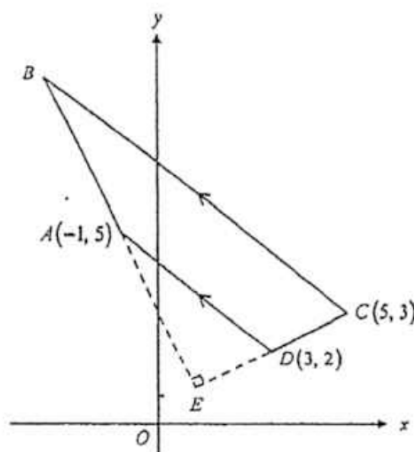
The line AB is parallel to the y -axis and points C and D are $(15, 0)$ and $(0, 8)$ respectively

- Show that coordinates of A is $(-2, 0)$ [2]
- Find the equation of the line that passes through A and perpendicular to the line BC [2]
- Find the coordinates of E if the ratio of area ABC to the area of ACE is given to be $2 : 1$ [5]
- Given that $ABFE$ is a kite, find the area of $ABFE$ [2]

Credit: S4 CHIJ SJC P1/2015 PRELIM Qn 10

2. Solutions to this question by accurate drawing will not be accepted

The diagram shows the trapezium $ABCD$ in which BC is parallel to AD while BA produced is perpendicular to CD produce at point E . The point A is $(-1, 5)$, C is $(5, 3)$ and D is $(3, 2)$



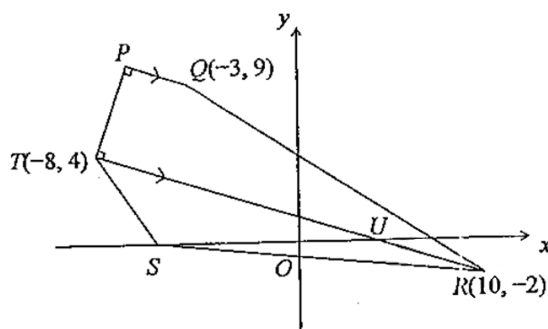
- Show that the coordinates of B are $(-3, 9)$ [6]
- Find the area of trapezium $ABCD$ [2]
- Find the coordinates of E given that [3]

$$\frac{\text{Area of } \triangle AED}{\text{Area of } \triangle BEC} = \frac{1}{4}$$

Credit: S4 VS P1/2015 PRELIM Qn 11

3. Solutions to this question by accurate drawing will not be accepted

The diagram shows a pentagon $PQRST$ in which PQ is parallel to TR and PT is perpendicular to PQ and TR . The coordinates of Q , R and T are $(-3, 9)$, $(10, -2)$ and $(-8, 4)$ respectively



Find

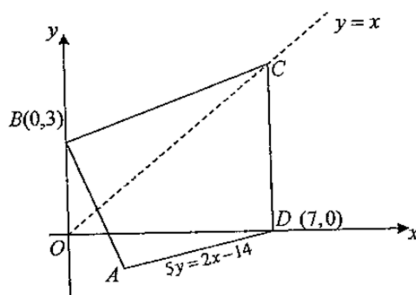
- (a) the coordinates of U [2]
- (b) the coordinates of P [4]
- (c) the ratio of the area of $\triangle RSU$ to the area of $\triangle STU$ [3]
- (d) the area of trapezium $PQRT$ [2]
- (e) W is a point such that $PQRW$ is a parallelogram. Find [2]

$$\frac{\text{Area of parallelogram } PQRW}{\text{Area of trapezium } PQRT}$$

Credit: S4 CCHS(Y) P2/2016 PRELIM Qn 7

4. Solutions to this question by accurate drawing will not be accepted

The diagram shows a quadrilateral $ABCD$ in which the point $B(0, 3)$ and the point $D(7, 0)$. The equation of the line AD is $5y = 2x - 14$ and C lies on the line $y = x$. The line CD is parallel to the y -axis



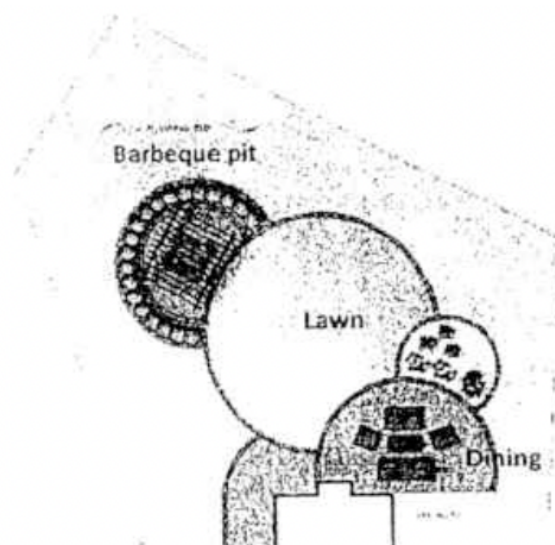
Given that A lies on the perpendicular bisector of BD

- (a) find the coordinates of A and of C [4]
- (b) find the area of the quadrilateral $ABCD$ [2]
- (c) explain clearly whether or not the quadrilateral $ABCD$ is a kite [3]

Credit: S4 CWSS P2/2016 PRELIM Qn 10

1.9 Further Coordinate Geometry

1. A landscaping company has been tasked to design the backyard for a client. The design is made up of overlapping circles as shown below



The circular lawn in the centre will be the focus point of the design and a barbeque pit will be constructed on one side of the lawn. The following model can be sketched on the Cartesian plane, and the circular lawn can be modelled by the equation of a circle

$$x^2 + y^2 + 2x - 6y - 15 = 0$$

- (a) Show why this model suggests that the radius of the lawn is 5 m [2]
- (b) A lamp post is positioned at a point $P(-5, 8)$ in the pit area. Determine, with working, if P lies inside or outside of the lawn [3]
- (c) Two dustbins, at Q and R , will be placed on the circumference of the lawn such that $Q(-4, -1)$ and QR is the diameter of the lawn. Find the equation of the tangent to the lawn at R [6]

Credit: **S4 AHS P2/2015 PRELIM Qn 11**

2. (a) A circle C_1 has an equation given by, where k is a positive constant [4]

$$x^2 + y^2 - 2kx + 2y + 1 = 0$$

Given that C_1 has a radius of 2 units, find the value of k

- (b) The centre of a circle C_2 lies on the line $y = 2x + 2$. Given that C_2 passes through the points $(3, 2)$ and $(0, -1)$, find the equation of C_2 [5]

Credit: **S4 HIHS P2/2015 PRELIM Qn 10**

3. (a) The equation of a circle is [5]

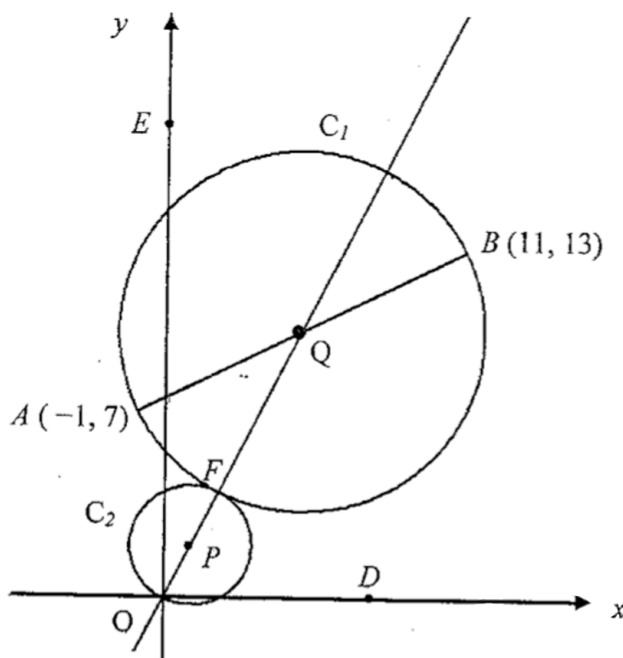
$$x^2 + 2x + 4y = 20 - y^2$$

Given that $A(2, 2)$ is a point on the circle, find the equation of the tangent to the circle at A

- (b) $A(0, 2)$, $B(9, 3)$ and $C(1, -7)$ are three points on a circle
 - (i) Show that BC is a diameter of the circle [4]
 - (ii) Find the equation of the circle [3]

Credit: **S4 CHIJ Sec P2/2016 PRELIM Qn 8**

4. The diagram below shows two circles C_1 and C_2 touching each other at point F . C_1 has centre at Q and C_2 has centre at P . The points $A(-1, 7)$ and $B(11, 13)$ lie on C_1 , and AB is the diameter of C_1 . The points O , P and Q lie on a straight line



- Find the equation of C_1 [3]
- Find the equation of the tangent to the 2 circles at F , given that the point F is $(2, 4)$ [3]
- If the coordinates of P is $(1, 2)$, determine whether a point $(1, 5)$ lies inside, outside or on circle C_2 [2]
- A third circle C_3 is drawn with DE as its diameter, where D and E are points on the x and y axis respectively. State whether the origin O lies on C_3 . Explain your answer [1]

Credit: **S4 BHSS P2/2016 PRELIM Qn 11**

5. (a) Find the equation of the two lines that are tangents to the circle and passes through point $(-3, 0)$ [6]

$$(x - 2)^2 + y^2 = 5$$

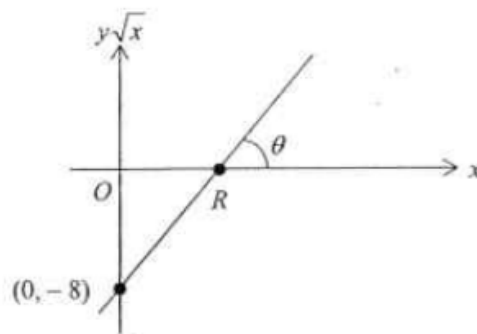
- Find the coordinates of the intersection of the circle with the tangent lines [3]
- State the number of intersections between the line $y = \frac{1}{4}(x + 3)$ and the circle. Justify your answer without finding the actual intersection points [2]

Credit: **S4 CGS P2/2016 PRELIM Qn 11**

1.10 Linear Law

1. The diagram shows part of a straight line graph drawn to represent the equation, where k is a constant

$$y + \frac{k}{\sqrt{x}} = 5\sqrt{x}$$



Given that the line passes through the point $(0, -8)$ and makes an angle of θ with the x -axis at point R , where $0^\circ < \theta < 90^\circ$, find

- (a) the value of k and of θ [4]
 (b) the coordinates of R [2]

Credit: **S4 PLMGS P1/2015 PRELIM Qn 5**

2. (a) It is known that x and y are related by the equation, where a and k are non-zero constants [3]

$$ax^2 + ky^2 - 120 = 0$$

Explain how the value of a and k may be obtained from a suitable straight line graph

- (b) A straight line graph is obtained by plotting $\frac{1}{y}$ against x . Given that the graph passes through the point $(\sqrt{3}, 1)$ and makes an angle of 60° with the line $y = 1$, express y in terms of x [4]

Credit: **S4 CGS P1/2016 PRELIM Qn 8**

3. The amount of expenditure, $\$y$, incurred by a textile company is related to $\$x$, the amount of sales generated. The variables x and y are related by the formula, where a and k are constants

$$y = 10^k x^a$$

The following table shows corresponding values of x and of y

x	6	35	234	1995	6310
y	148	295	628	1480	2344

- (a) Plot $\lg y$ against $\lg x$ for the given data and draw a straight line graph [3]
 (b) Use your graph to estimate the value of a and of k [4]
 (c) Estimate the amount of expenditure incurred when the sales generated is \$4000 [2]
 (d) Draw a straight line on the same axes to estimate the amount of sales to be generated in order for the textile company to breakeven [2]

Credit: **S4 MSHS P2/2016 PRELIM Qn 11**

4. The table shows experimental values of two variables x and y

x	2	3	4	6	10
y	3.24	5.79	9	17.05	38.43

It is known that x and y are related by the equation for $x > 0$, where a and b are constants

$$\frac{y-b}{x} = a\sqrt{x} - 1$$

- (a) Using a scale of 1 cm to 2 units on the horizontal axis and 2 cm to 5 units on the vertical axis, draw a straight line graph of $(x+y)$ against $x\sqrt{x}$ [3]

- (b) Use your graph to estimate, to 2 decimal places, the value of a and of b [4]

- (c) On the same diagram, draw a straight line representing the equation [3]

$$y + x + 2x\sqrt{x} = 36$$

- (d) Hence, find the value of x that satisfies the equation [1]

$$(a+2)x\sqrt{x} = 36 - b$$

Credit: **S4 MGS P1/2016 PRELIM Qn 11**

5. An experiment to find the constant acceleration, $a \text{ m/s}^2$, of an electric toy car moving in one direction requires students to measure the speed, $v \text{ m/s}$ from the speedometer when distance, $s \text{ m}$ varies. The table below shows the experimental values of v and s , which are connected by the equation, where p is a constant

$$v = \sqrt{e^p + 2as}$$

s	4.167	17.5	37.5	80
v	3	5	6	10

- (a) (i) Plot v^2 against s and draw a straight line graph. [3]

- (ii) Hence, determine which value of v , in the table above, is the incorrect recording. [1]

- (iii) Using your graph to estimate the correct v value [1]

- (b) Use your graph to estimate the value of a and of p [3]

- (c) Explain what does the value e^p represent [1]

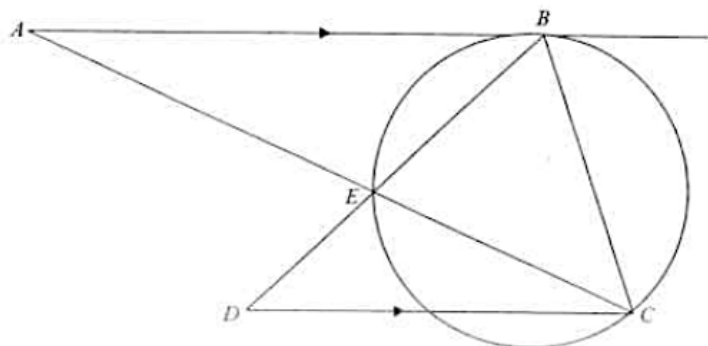
- (d) By drawing a suitable straight line on your graph, solve [2]

$$s = \frac{120 - 2e^p}{4a + 3}$$

Credit: **S4 NCHS P2/2016 PRELIM Qn 6**

1.11 Proofs of Plane Geometry

1. In the diagram, AB is a tangent to the circle at the point B , BED is straight and AB is parallel to DC . The points A , E and C lie on a straight line and $AE : EC = 2 : 1$



- (a) Prove that

$$\angle BCE = \angle BDC$$

[2]

- (b) **Hence**, show that $\triangle BCE$ is similar to $\triangle BDC$

[2]

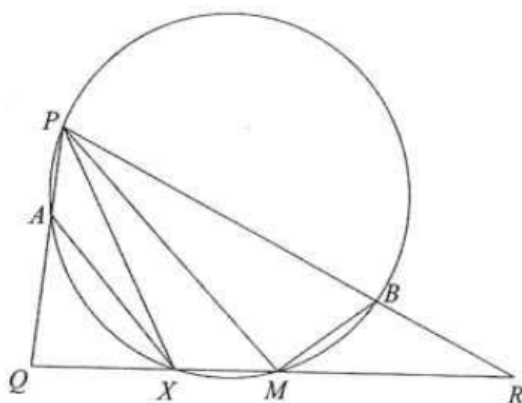
- (c) Prove that

$$3AE \times CD = AB \times AC$$

[3]

Credit: **S4 ACS(I) P2/2015 PRELIM Qn 5**

2. In the triangle PQR , M is the mid-point of QR and PX bisects $\angle QPR$. The circle passing through P , X and M , cuts PQ and PR at A and B respectively



- (a) Explain why

$$\angle PBM + \angle PXM = 180^\circ$$

[1]

- (b) Show that $\triangle RBM$ is similar to $\triangle RXP$

[3]

- (c) Given that $\triangle QXA$ is also similar to $\triangle QPM$ and

[4]

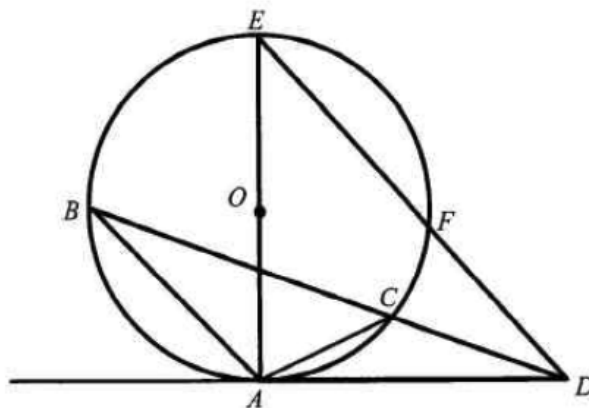
$$\frac{PR}{RX} = \frac{PQ}{QX}$$

show that

$$RB = QA$$

Credit: **S4 FMS(S) P2/2015 PRELIM Qn 6**

3. In the diagram, $\triangle ABC$ is inscribed in the circle with centre O .



The tangent at A meets the line EF and BC produced at D

- (a) Prove that $\triangle ADC$ and $\triangle BDA$ are similar

[2]

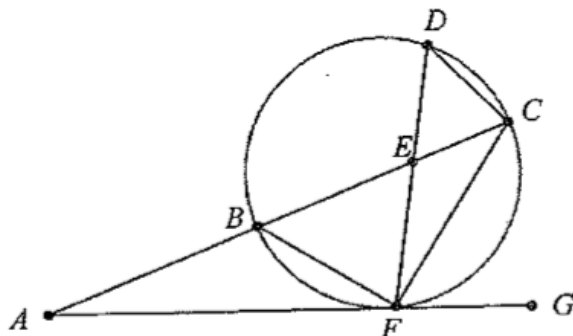
- (b) Prove that

[3]

$$BD \times CD = DE^2 - AE^2$$

Credit: S4 NCHS P1/2016 PRELIM Qn 10

4. In the figure, BC is a diameter of the circle



ABC is a straight line and AG is a tangent to the circle at point F . The line DF intersects BC at point E and $3EC = 2EB$

- (a) Prove that $\triangle ABF$ and $\triangle AFC$ are similar

[3]

- (b) Show that

[2]

$$AF \times FC = BF \times AC$$

- (c) Given that $\triangle DEC$ and $\triangle BEF$ are similar, prove that

[3]

$$EF \times ED = \frac{6}{25} BC^2$$

Credit: S4 NBSS P2/2016 PRELIM Qn 5

1.12 Differentiation

1. A function is given by

$$y = \frac{9x - 3b}{4x - 1} \quad \text{where } x \neq a, \quad x > 0$$

- (a) State the value of a [1]
 (b) Determine the range of values of b if y is an increasing function [3]
 (c) Given that $b = 3$, and that x and y vary with time t , find the value(s) of x if [3]

$$\frac{dy}{dt} = 12 \left(\frac{dx}{dt} \right)$$

Credit: **S4 ANDSS P1/2015 PRELIM Qn 12**

2. The voltage,
- V
- , in volts, of an electrical signal in an electrical system is given by the following formula, where
- t
- is in seconds

$$V = 4 \sin \pi t$$

- (a) Find the exact rate of change of voltage after $\frac{1}{4}$ seconds have elapsed [2]
 (b) Find the exact time when the rate of change of voltage is $2\pi\sqrt{3}$ volts per second for $0 < t < 4$ [3]
 (c) Given that the current (I in amperes) supplied to the system is governed by the equation [2]

$$I = \frac{V}{5}$$

find the rate of change of current when the rate of change of voltage is 2 volts per second

Credit: **S4 AHS P1/2015 PRELIM Qn 9**

3. A function has an equation where

$$f(x) = \frac{\ln(4-x)}{x-4} \quad x < 4$$

- (a) Explain why the condition of $x < 4$ is necessary [1]
 (b) Obtain an expression for $f'(x)$ [3]
 (c) Showing full working, determine whether f is decreasing for $x < 4 - e$ [3]

Credit: **S4 TLSS P2/2015 PRELIM Qn 2 (MODIFIED)**

4. The equation of a curve is

$$f(x) = x^3 \ln x$$

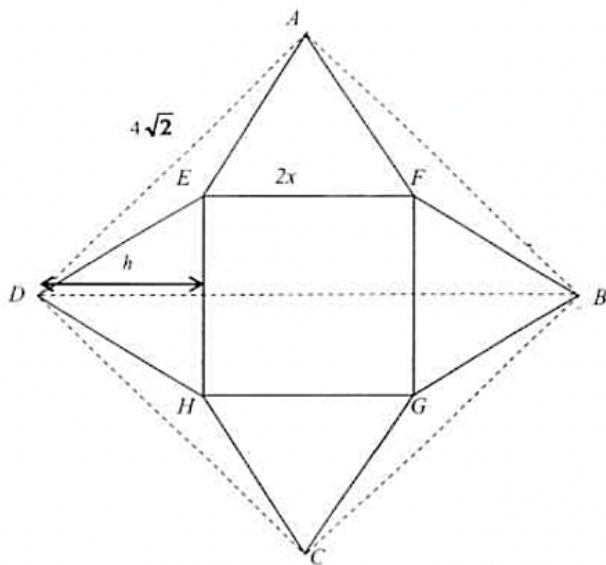
- (a) Show that the curve $f(x)$ has only one stationary point [3]
 (b) Find the x -coordinate of the stationary point of the curve in exact form [2]
 (c) Prove that the value of $f''(x)$ at the stationary point is [2]

$$\frac{3}{\sqrt[3]{e}}$$

- (d) What does the result of part (c) imply about the stationary point [1]

Credit: **S4 BHSS P2/2016 PRELIM Qn 4 (MODIFIED)**

5. The diagram above shows a square piece of cardboard paper $ABCD$ of side $4\sqrt{2}$ metres



Triangles AED , AFB , DHC and BGC are cut off leaving a figure in the shape of a square $EFGH$ of sides $2x$ metres and 4 identical isosceles triangles attached to the sides. The height of each triangle is h metres. Mark wants to fold the paper to make a pyramid with $EFGH$ as the base

- (a) Show that $h = 4 - x$ [2]

- (b) Show that the volume of the pyramid, $V \text{ m}^3$, is given by [4]

$$V = \frac{8}{3}x^2\sqrt{4 - 2x}$$

- (c) **Hence**, find the maximum volume of the pyramid. (Proof of maximum is not required) [4]

Credit: **S4 ACS(B) P1/2015 PRELIM Qn 1**

6. The equation of a curve is

$$y = (x + k)^2$$

- (a) Show that the equation of the tangent to the curve where $x = 2k$ is [5]

$$y + 3k^2 = 6kx$$

This tangent meets the x -axis at P and the y -axis at Q . The midpoint of PQ is M

- (b) Show that M lies on the curve [4]

$$y + 24x^2 = 0$$

Credit: **S4 GMS(S) P1/2016 PRELIM Qn 3**

1.13 Integration

1. A curve is such that

$$\frac{d^2y}{dx^2} = 16e^{-4x}$$

[6]

Given that $\frac{dy}{dx} = 3$ when $x = 0$ and that the curve passes through the point $(2, e^{-8})$, find the equation of the curve

Credit: **S4 HIHS P1/2015 PRELIM Qn 6**

2. Given that

$$\int_{-1}^3 [f(x) + 1] \, dx = 8$$

evaluate

(a)

$$\int_{-1}^3 f(x) \, dx$$

[2]

(b)

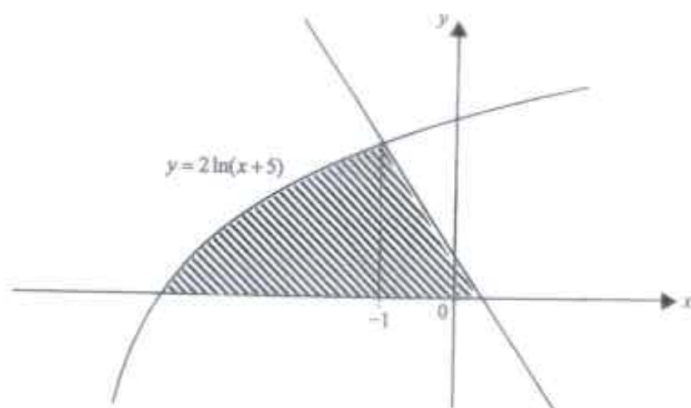
$$\int_2^3 [f(x) + 1] \, dx - \int_2^{-1} [f(x) + 1] \, dx$$

[2]

Credit: **S4 TKSS P1/2015 PRELIM Qn 3**

3. The diagram below shows the curve with following equation and the normal to the curve at $x = -1$

$$y = 2 \ln(x + 5)$$



- (a) Show that the equation of the normal to the curve at $x = -1$ is

[3]

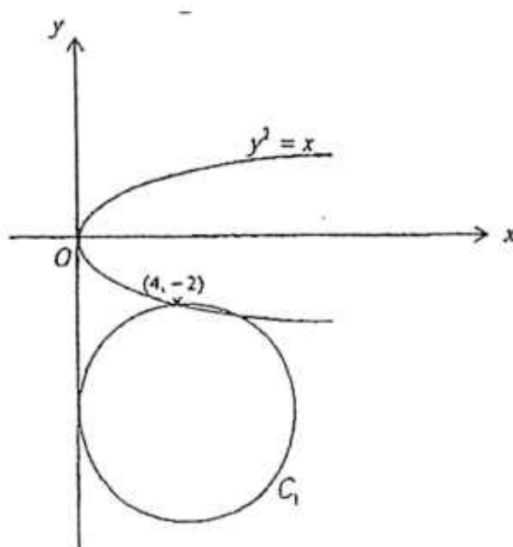
$$y = -2x + 4 \ln 2 - 2$$

- (b) Find the area of the shaded region bounded by the curve, the normal to the curve at $x = -1$ and the x -axis, leaving your answer in 2 decimal places

[6]

Credit: **S4 CGS P2/2016 PRELIM Qn 9**

4. The diagram shows a circle C_1 with centre $(4, -6)$



A curve $y^2 = x$ and the circle, C_1 , have the y -axis as the common tangent. Both curves intersect at the point $(4, -2)$

- Write down the radius of the circle C_1 [1]
- Hence**, the equation of C_1 [1]
- Find the area bounded by the curve $y^2 = x$, the circle C_1 and the y -axis [3]
- A second circle, C_2 , is the reflection of the circle, C_1 , in the line $y = 2$. Write down the equation of the second circle, C_2 , in the form [2]

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

Just a note

Although power functions $y^2 = x$ is no longer in the **4049 Syllabus**, the question is still doable

Credit: **S4 TSS P1/2015 PRELIM Qn 5**

5. Given that

$$\int_m^6 \frac{x-2}{2x^2-x-6} dx = \frac{1}{2} \ln \frac{5}{3}$$

- state the value(s) of x for the integral to be undefined [2]
- find the value of m [4]

Credit: **S4 FHSS P2/2016 PRELIM Qn 1**

1.14 Differentiation & Integration

1. (a) Given that

$$y = x^2 \ln x^3$$

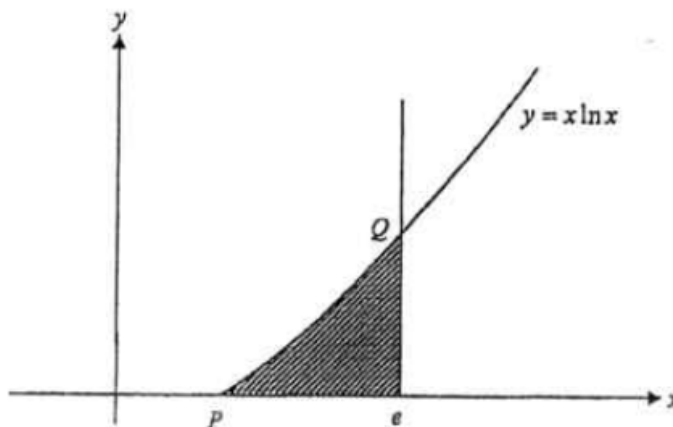
[3]

show that

$$\frac{dy}{dx} = 3x(1 + 2 \ln x)$$

- (b) The diagram shows part of the curve cutting the x -axis at point P

$$y = x \ln x$$



The line $x = e$ intersects the curve at point Q

- (i) Find the x -coordinate of P [2]
 (ii) By using the result from part (i), show that the area of the shaded region bounded by the x -axis, the line $x = e$ and the curve is [4]

$$\text{Area} = \frac{1}{4}(e^2 + 1)$$

Credit: **S4 CHIJ SJC P2/2015 PRELIM Qn 9**

2. It is given that $f(x)$ is such that

$$f'(x) = \sin 2x + \cos 3x$$

[6]

Given also that

$$f\left(\frac{\pi}{6}\right) = 0$$

show that

$$f''(x) + 9f(x) = -\frac{3}{4} - \frac{5}{2} \cos 2x$$

Credit: **S4 SCSS P1/2015 PRELIM Qn 7**

3. It is given that $f(x)$ is such that

$$f'(x) = \frac{1}{2x-5} - \frac{4}{(2x-5)^2}$$

[7]

Given also that

$$f(3) = 1.75$$

show that

$$8f(x) - (2x-5)^2 f''(x) = \ln(2x-5)^4$$

Credit: **S4 CCHS(M) P2/2016 PRELIM Qn 10(b)**

4. (a) Find the range of values of x for which the curve is a decreasing function [4]

$$y = x^3 e^{1-2x}$$

- (b) (i) Given that [3]

$$y = [\ln(3 - 4x)]^2$$

show that the following where k is a constant to be determined

$$\frac{dy}{dx} = \frac{k \ln(3 - 4x)}{3 - 4x}$$

- (ii) Hence, evaluate [4]

$$\int_{-2}^{-1} \frac{2 + 3 \ln(3 - 4x)}{3 - 4x} dx$$

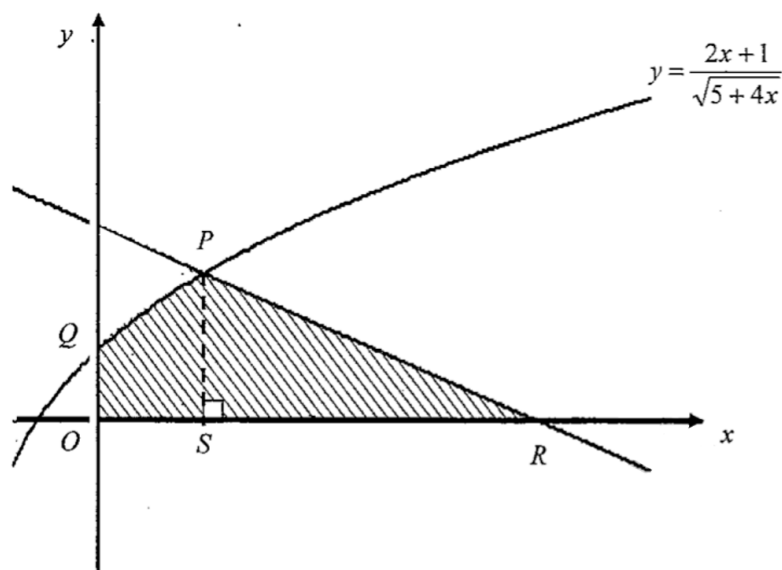
Credit: S4 TSS P2/2015 PRELIM Qn 10 (MODIFIED)

5. (a) Show that [3]

$$\frac{d}{dx} [(x - 1)\sqrt{5 + 4x}] = \frac{6x + 3}{\sqrt{5 + 4x}}$$

- (b) The diagram shows part of the curve

$$y = \frac{2x + 1}{\sqrt{5 + 4x}}$$



The line PR is a normal to the curve at P . Q is the point where the curve cuts the y -axis and S is a point directly below P

- (i) Given that the x -coordinate of P is 1, find the equation of the line PR [4]
 (ii) Without calculating the area under the curve from $x = 0$ to $x = 1$, explain briefly why [2]

$$\int_0^1 \frac{2x + 1}{\sqrt{5 + 4x}} dx > \frac{1}{2} \left(1 + \frac{1}{\sqrt{5}} \right)$$

- (iii) Find the area of the shaded region [3]

Credit: S4 BHSS P2/2016 PRELIM Qn 9

1.15 Kinematics

1. A particle, travelling in a straight line, passes a fixed point O on the line with a speed of 2 ms^{-1} . The acceleration, $a \text{ ms}^{-2}$, of the particle, $t \text{ s}$ after passing O , is given by

$$a = -4e^{-t}$$

- (a) Show that the particle comes instantaneously to rest when $t = -\ln \frac{1}{2}$ [4]
 (b) Find the total distance travelled by the particle between $t = 0$ and $t = 2$ [6]
 (c) Find the average velocity of the particle during the first 2 seconds [1]

Credit: **S4 SCSS P2/2015 PRELIM Qn 11**

2. A particle Q passes a fixed point B and moves in a straight line such that, t seconds after leaving B , its velocity, $v \text{ m/s}$, is given by

$$v = 2 \cos^2 t - 1$$

Find

- (a) the acceleration of Q when $t = 2$ [2]
 (b) the time when the particle is instantaneous rest for $0 \leq t \leq 2$ [2]
 (c) the total distance travelled by Q in the first 2 seconds [3]

Credit: **S4 FHSS P1/2016 PRELIM Qn 8**

3. The velocity, $v \text{ m/s}$, of a particle, P , moving in a straight line is given by the following, where t is the time in seconds after the start of motion

$$v = 3t^2 + pt + q$$

At $t = 0$, the displacement of the particle from O is 3 m. Given also that when $t = 2$, the displacement of the particle from O is 23 m and the acceleration of the particle is -6 ms^{-2}

- (a) find the value of p and of q [7]
 (b) explain with clear working whether P will return to its starting point [5]

Credit: **S4 SCGS P2/2016 PRELIM Qn 11**

4. A particle moves in a straight line such that, t seconds after leaving a fixed point O , its velocity, $v \text{ m/s}$, is given by

$$v = 15 - e^{-3t}$$

- (a) Write down the initial velocity of the particle [1]
 (b) If t becomes very large, what value will v approach? Explain your answer clearly and its significance [2]
 (c) Find the acceleration of the particle when $t = 3$, giving your answer in cm s^{-2} correct to 3 decimal places [2]
 (d) Find the distance travelled by the particle in the first 4 seconds of its journey, giving your answer correct to 2 decimal places [2]

Credit: **S4 SMSS P1/2016 PRELIM Qn 10**

2 Final Answers

2.1 Quadratic Equations & Inequalities

1. (a) (i) $p \leq 4$ and $p \geq 12$
(ii) $p = 4$ or $p = 12$
(iii) $p > 6$
(b) Shown
2. (a) (i) Shown
(ii) $-4 < x < 7$
(b) $k > -\frac{1}{24}$
3. (a) $x < 2\frac{1}{2}$ and $x > 3$
(b) (i) Shown
(ii) $k = -2$
(c) $p > 0$ and $pq > 4$
4. (a) Shown
(b) Shown

2.2 (Indices) & Surds

1. $(4 - \sqrt{5})$ cm
2. (a) $k = 10$ $n = 1$
(b) (i) $(4 - \sqrt{2})$ cm
(ii) $(15 - 10\sqrt{2})$ cm
(iii) $\frac{5}{2}(\sqrt{2} - 1)$ cm³
3. (a) $(4 + 2\sqrt{3})$ cm
(b) $(28 + 16\sqrt{3})$ cm²
4. (a) $15 + 6\sqrt{5}$
(b) (i) $846 - 26\sqrt{5}$
(ii) $(3970 + 1562\sqrt{5})$ cm³
5. (a) Shown
(b) $\frac{1}{17}(95 - 64\sqrt{2})$
(c) Shown

2.3 Polynomials

1. (a) Shown
(b) Graph
2. (a) $p = 3$ $q = 0$
(b) $f(x) = (x + 2)(x^2 - x + 2)$
3. (a) $m = 3$
(b) (i) $k = 2\frac{1}{2}$
(ii) 45
4. (a) $A = -2$
(b) $x = 1$ or $x = -\frac{3 \pm \sqrt{5}}{2}$
5. (a) Shown
(b) 5

2.4 Partial Fractions

1. (a) $\frac{3}{x} - \frac{5}{x^2} + \frac{3}{1+x}$
(b) $3 \ln\left(\frac{x}{1-x}\right) + \frac{5}{x} + c$
2. (a) $\frac{1}{x+2} + \frac{2}{(x+2)^2} + \frac{2}{x-2}$
(b) (i) $\ln(x+2) - \frac{2}{x+2} + 2 \ln(x-2) + c$
(ii) Shown
3. (a) $\frac{2}{3x-1} - \frac{3}{x+4}$
(b) 0.0213
4. (a) (i) Shown
(ii) $f(x) = (3x-1)(x+2)^2$
(b) $\frac{2}{3x-1} + \frac{1}{x+2} - \frac{5}{(x+2)^2}$
(c) $\frac{2}{3} \ln(3x-1) + \ln(x+2) + \frac{5}{x+2} + c$

2.5 Binomial Theorem

1. 10
2. (a) (i) $1 - 8x + 28x^2 - 56x^3 + \dots$
(ii) -560
(b) (i) $\binom{6}{r} (2)^{6-r} \left(-\frac{1}{3}\right)^r x^{6-4r}$
(ii) No constant term
(iii) -48
3. (a) $13\frac{1}{8}$
(b) (i) $a = 288$ $b = 84$
(ii) 9132
4. (a) (i) $1 + 36x + 540x^2 + \dots$
(ii) $1 - 6kx + 15k^2x^2 + \dots$
(b) $15k^2 - 216k + 540$
(c) 2
5. (a) $\binom{n}{r} \left(\frac{x}{2}\right)^{n-r} \left(-\frac{k}{x^2}\right)^r$
(b) Shown
(c) 10
(d) 36750

2.6 Exponential & Logarithms

1. (a) $x = \frac{2}{3}$
(b) (i) Graph
(ii) $y = x - 3$
2. (a) $a = 3$ $b = 5$
(b) $\ln \frac{1}{2}$
3. (a) $V = 1500(2)^{\frac{x}{5}}$
(b) Graph
(c) 46.2 years
4. (a) 2
(b) $1\frac{3}{10}$
5. (a) 5
(b) 10
(c) 1000

2.7 Trigonometry

1. (a) (i) $y = 0^\circ$ $y = 109.5^\circ$ $y = 250.5^\circ$ $y = 360^\circ$
 (ii) Shown
 (b) 39
2. (a) Graph
 (b) (i) 3
 (ii) Day 2
3. (a) Shown
 (b) 0.0658 1.11 2.16
4. (a) (i) Shown
 (ii) $\theta = 1.25$ $\theta = \frac{3\pi}{4}$ $\theta = 4.39$ $\theta = \frac{7\pi}{4}$
 (b) (i) Shown
 (ii) $A = \frac{25\sqrt{5}}{8} \sin(\sin 2\theta - 26.6^\circ) - \frac{25}{8}$
 (iii) $\frac{25\sqrt{5} - 25}{8}$
5. (a) Shown
 (b) Shown
 (c) $c = d = 1$
 (d) Shown

2.8 Coordinate Geometry

1. (a) Shown
 (b) $y = \frac{15}{8}x + \frac{15}{4}$
 (c) $E\left(23\frac{1}{2}, -4\frac{8}{15}\right)$
 (d) $231\frac{1}{5}$ units²
2. (a) Shown
 (b) 15 units²
 (c) $E(1, 1)$
3. (a) $U(4, 0)$
 (b) $P(-6, 10)$
 (c) $\frac{1}{2}$
 (d) 70 units²
 (e) $\frac{2}{7}$
4. (a) $A(2, -2)$ $C(7, 7)$
 (b) 39 units²
 (c) Not a kite

2.9 Further Coordinate Geometry

- Shown
 - Outside
 - $y = -\frac{3}{4}x + \frac{17}{2}$
- 2
 - $x^2 + (y - 2)^2 = 9$
- $y = -\frac{3}{4}x + \frac{7}{2}$
 - Shown
 - $(x - 5)^2 + (y + 2)^2 = 41$
- $(x - 5)^2 + (y - 10)^2 = 45$
 - $y = -\frac{1}{2}x + 5$
 - Outside
 - Yes, the origin lies on C_3
- $y = -\frac{1}{2}x - \frac{3}{2}$ or $y = \frac{1}{2}x + \frac{3}{2}$
 - $(1, 2)$ or $(1, -2)$
 - 2 points of intersection

2.10 Linear Law

Note:

Questions 3(b),(c),(d), 4(b),(d) & 5(b),(d) are suggested answers, range is ± 0.2

- $k = -8$ $\theta = 78.7^\circ$
 - $R\left(1\frac{3}{5}, 0\right)$
- Explain
 - $y = \frac{1}{\sqrt{3} - 2}$
- Graph
 - $a = 0.396$ $k = 1.86$
 - \$3981.07
 - \$1174, 90
- Graph
 - $a = 1.51$ $b = 0.97$
 - $(x + y) = -2(x\sqrt{x}) + 36$
 - 4.64
- Graph
 - 6 m/s
 - 7 m/s
 - $a = 0.6$ $p = \ln 4$
 - Square of initial velocity of the electric toy car
 - 20.7 m

2.11 Proofs in Plane Geometry

1. Prove
2. Prove
3. Prove
4. Prove

2.12 Differentiation

1. (a) $\frac{1}{4}$
(b) $b > \frac{3}{4}$
(c) $\frac{5}{8}$
2. (a) $\frac{4\pi}{\sqrt{2}}$ v/s
(b) $t = \frac{1}{6}$ sec $t = 1\frac{5}{6}$ sec $t = 2\frac{1}{6}$ sec $t = 3\frac{5}{6}$ sec
(c) $\frac{2}{5}$ amperes/sec
3. (a) Explain
(b) $\frac{1 - \ln(4 - x)}{(x - 4)^2}$
(c) Shown
4. (a) Shown
(b) $\frac{1}{\sqrt[3]{e}}$
(c) Shown
(d) Minimum point
5. (a) Shown
(b) Shown
(c) 6.11 m^3
6. (a) $y = 6kx - 3k^2$
(b) Shown

2.13 Integration

1. $y = e^{-4x} + 7x - 14$
2. (a) 4
(b) 8
3. (a) Shown
(b) 7.01 units^2
4. (a) 4 units
(b) $(x - 4)^2 + (y + 6)^2 = 16$
(c) 6.10 units^2
(d) $x^2 + y^2 - 8x - 20y + 100 = 0$
5. (a) $x = 2$ or $x = -\frac{3}{2}$
(b) 3

2.14 Differentiation & Integration

1. (a) Shown
(b) (i) 1
(ii) Shown
2. Shown
3. Shown
4. (a) $x > 1\frac{1}{2}$
(b) (i) 8
(ii) 0.962
5. (a) Shown
(b) (i) $y = -x + 2$
(ii) Shown
(iii) $\left(\frac{1}{3}\sqrt{5} + 1\right) \text{ units}^2$

2.15 Kinematics

1. (a) Shown
(b) 1.77 m
(c) -0.271 m/s
2. (a) 1.51 m/s^2
(b) $\frac{\pi}{8} \text{ sec}$
(c) 1.08 m
3. (a) $p = -18$ $q = 24$
(b) Never return to the starting point
4. (a) 14 m/s
(b) 15 m/s Explain
(c) 0.037 cm/s^2
(d) 59.67 m