Name:	Level/Subject: **4049 Sec 4 A-Math**
Material: **May Practice Questions 2022**	Centre: **Overmugged**

Instructions

- Answer all questions
- If working is needed for any question it must be shown with the answer
- Omission of essential working will result in loss of marks
- You are expected to use a scientific calculator to evaluate explicit numerical expressions
- If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures
- Give answers in degrees to one decimal place
- For π, use either your calculator value of 3.142, unless the question requires the answer in terms of π
- A copy of the formula list is provided for you on the next page

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Question Source

All questions are sourced and selected based on the known abilities of students sitting for the 'O' Level A-Math Examination. All questions compiled here are from **2009 - 2021 School Mid-Year / Prelim Papers**. Questions are categorised into the various topics and range in varying difficulties. If questions are sourced from respective sources, credit will be given when appropriate.

How to read:

[S4 ABCSS P1/2011 PRELIM Qn 1]

Secondary 4, ABC Secondary School, Paper 1, 2011, Prelim, Question 1

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This question paper consists of 33 printed pages including the cover page

Contents

List of Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1}b + \binom{n}{2} a^{n-2}b^2 + ... + \binom{n}{r} a^{n-r}b^r + ... + b^n$$

where n is a positive integer and

$$\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)...(n-r+1)}{r!}$$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\text{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

1 Questions

1.1 Quadratic Equations & Inequalities

1. (a) Given that $p > \frac{1}{3}$, explain why the equation has no real solutions [4]

$$3^{2x+1} = 6\left(3^{x-1}\right) - p$$

(b) Find the **exact** range of values of the constant a for which the line intersects the curve at 2 distinct points [5]

$$y = 2x - \frac{a^2}{2}$$

$$y = x^2 - ax - 4$$

Credit: **S4 CGSS P2/2021 PRELIM Qn 2**

2. (a) Find the range of values of p such that the following lies entirely above the x-axis [4]

$$y = px^2 - 4x + p$$

(b) Explain clearly why the line will intersect the curve at 2 distinct points for all real values of k [5]

$$y = x + 2k$$

$$2y^2 - x^2 = 8$$

Credit: **S4 CHIJ Sec P1/2020 PRELIM Qn 8**

3. (a) Express $x^2 - x + 1$ in the form $(x - p)^2 + q$, where p and q are constants [2]

(b) Show that the curve will cut the x-axis at two distinct points for all real values of p [3]

$$y = x^2 - 2px + p - 1$$

Credit: **S4 CCHS(M) P2/2020 PRELIM Qn 4**

4. (a) Find the range of values of x for which [3]

$$-\frac{4}{3x^2 + 14x - 5} < 0$$

(b) Find the range of values of c for which the line intersects the curve at 2 distinct points [3]

$$x + y = c$$

$$y^2 = 2x + 3$$

Credit: **S4 HYSS P1/2020 PRELIM Qn 3**

1.2 (Indices) and Surds

1. (a) Find the value of 15^n given that [3]

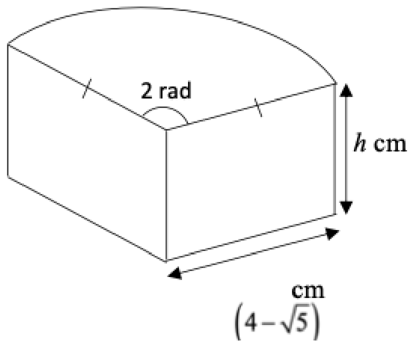
$$3^{n+2} - 3^n = \frac{5^{n+1}}{25^n}$$

(b) **Without using a calculator**, find the roots of the equation in the form $\dfrac{a + b\sqrt{c}}{4}$ [3]

$$x\sqrt{80} = \sqrt{20} - x\sqrt{48}$$

Credit: **S4 BPGHS P1/2020 EOY Qn 7**

2. A prism with volume $\left(50\sqrt{5} - 101\right)$ cm^3 has a cross-sectional area of the sector with radius $(4 - \sqrt{5})$ cm, angle of 2 radians and a height of h cm [4]



Without using a calculator, express h in the form $\left(a + b\sqrt{5}\right)$ cm, where a and b are integers

Credit: **S4 HSS P1/2020 PRELIM Qn 3**

3. A cone with base radius $(5+2\sqrt{3})$ cm and a slant height l cm has a curved surface area of $(51-3\sqrt{3})\pi$ cm^2. Without using a calculator, obtain an expression for l in the form $(a + b\sqrt{3})$, where a and b are integers [4]

Credit: **S4 JWSS P1/2020 PRELIM Qn 1**

4. **Without using a calculator**, find a and b given that, a and b are rational numbers [5]

$$\frac{\sqrt{7} - \sqrt{6}}{\sqrt{21} + \sqrt{2}} = a\sqrt{3} + b\sqrt{14}$$

Credit: **S4 NGHS P2/2020 PRELIM Qn 1**

1.3 Polynomials

1. The equation of a polynomial is

$$f(x) = 9x^3 - 6x^2 - 11x + 4$$

   (a) Factorise completely the polynomial $f(x)$ [3]

   (b) Sketch the graph of $f(x) = 9x^3 - 6x^2 - 11x + 4$ [2]

   (c) Find the range of values of x for which $f(x) \geq 0$ [2]

   Credit: **S4 CHIJ SNGS P1/2021 PRELIM Qn 6**

2. The roots of a cubic equation $F(x) = 0$ are -1, 2 and 5. When $F(x)$ is divided by $(x-3)$, the remainder is 30

   (a) Find the remainder when $F(x)$ is divided by $(x + 3)$ [4]

   (b) Solve the equation [2]

$$F\left(\sqrt{m}\right) = 0$$

   Credit: **S4 ZHSS P1/2021 PRELIM Qn 5**

3. The quartic (4th degree) polynomial $f(x)$ is such that the coefficient of x^4 is 3. Two of the roots of $f(x) = 0$ are -2 and 3. One of the factors of $f(x)$ is $x^2 - 3x - 1$

   (a) How many real solutions does $f(x) = 0$ have? Justify your answer [4]

   (b) Write an expression for $f(x)$ in descending powers of x [2]

   (c) Find the remainder when $f(x)$ is divided by $(2x + 1)$ [2]

   Credit: **S4 HYSS P2/2020 PRELIM Qn 6**

4. (a) Solve the equation [4]

$$2x^3 - 3x^2 - 3x + 4 = 0$$

   (b) It is given that $(x - 5)$ is a factor of $p(x) + 1$, where $p(x)$ is a polynomial. Find the remainder when $g(x) = 2x^3 - p(x) + 5$ is divided by $(x - 5)$ [2]

   Credit: **S4 NHHS P1/2020 PRELIM Qn 7**

5. The function, where p and q are constants, is exactly divisible by $\left(x^2 - 4\right)$

$$P(x) = 2x^4 + p\left(x^3 + x^2\right) + q(3x - 5)$$

   (a) Find the value of p and of q [5]

   (b) Find the remainder when $P(x)$ is divided by $(2x + 1)$ [2]

   (c) Determine, showing all necessary working, the number of real roots of the equation [3]

$$P(x) = 0$$

   Credit: **S4 NGHS P2/2020 PRELIM Qn 5**

1.4 Partial Fractions

1. (a) Express the following in partial fractions [5]

$$\frac{4x^3 + 5x^2 + x - 1}{x^2(x+1)}$$

(b) Hence, find [3]

$$\int \frac{4x^3 + 5x^2 + x - 1}{x^2(x+1)}\ dx$$

Credit: **S4 ZHSS P2/2021 PRELIM Qn 2**

2. (a) Express the following in partial fractions [4]

$$\frac{5x^2 + 4x - 3}{x^2(2x-1)}$$

(b) Hence, show that [5]

$$\int_1^5 \frac{5x^2 + 4x - 3}{x^2(2x-1)}\ dx = \frac{12}{5} + \ln 75$$

Credit: **S4 ACS(B) P2/2020 PRELIM Qn 7**

3. (a) Show that $(x-3)$ is a factor of $2x^3 - 13x^2 + 24x - 9$ [3]

(b) Express the following as the sum of three partial fractions [5]

$$\frac{5x^2 - 30x + 10}{2x^3 - 13x^2 + 24x - 9}$$

(c) Hence, find [4]

$$\int \frac{10x^2 - 60x + 20}{2x^3 - 13x^2 + 24x - 9}\ dx$$

Credit: **S4 CHIJ Sec P2/2020 PRELIM Qn 3**

4. (a) Factorise [1]

$$x^3 + 8$$

(b) A pyramid has

Base area: $\left(x^3 + 8\right)$ cm^2 Volume: $\left(x^3 + \frac{1}{3}x^2 + \frac{14}{3}x + 4\right)$ cm^3

(i) Show that the height of the pyramid can be expressed as the following, where A, B and C are constants [2]

$$A + \frac{x^2 + Bx + C}{x^3 + 8}$$

(ii) Using your results in part (a) and part (b)(i), express the height of the pyramid as partial fractions [4]

Credit: **S4 MSS P2/2020 PRELIM Qn 2**

1.5 Binomial Theorem

1. (a) Write down and simplify, in descending powers of x, the first three terms in the expansion of the following, where $n > 0$ [2]

$$\left(x^5 + \frac{2}{x^6}\right)^n$$

(b) When the third term of the expansion is divided by the second term, $\frac{8}{x^{11}}$ is obtained. Show that $n = 9$ [2]

(c) Using the value of n found in part (ii), and without expanding the expression in part (i), show that there is no constant term in the expansion [3]

Credit: **S4 ANDSS P1/2021 PRELIM Qn 4**

2. Consider the expansion, where a is a positive constant

$$\left(\frac{a^2}{\sqrt{x}} - \frac{\sqrt{x}}{a}\right)^8$$

Find, **in terms of a**,

(a) the term independent of x [2]

(b) the coefficient of x^2 in the expansion of [4]

$$\left(\frac{3x^4 - 4x^2}{x^2}\right)\left(\frac{a^2}{\sqrt{x}} - \frac{\sqrt{x}}{a}\right)^8$$

Credit: **S4 CGSS P1/2021 PRELIM Qn 4**

3. (a) Write down, and simplify, the first 4 terms in the expansion of $(1 + x)^7$ in ascending x [2]

(b) Write down the general term in the binomial expansion of [1]

$$\left(x^2 - \frac{2}{x^3}\right)^9$$

(c) Write down the power of x in this general term [1]

(d) **Hence**, or otherwise, determine the coefficient of x^3 in the expansion of [2]

$$(1 + x)^7 + \left(x^2 - \frac{2}{x^3}\right)^9$$

Credit: **S4 CCHS(M) P2/2020 PRELIM Qn 5**

4. (a) Given that the coefficient of x^3 in the following expansion is $\frac{595}{4}$ [5]

$$(3 - px)^5 + (2 + x)^6$$

find the value of p

(b) Calculate the coefficient of x^3 in the expansion of [4]

$$\left(x^2 - 2x\right)^2 (2 + x)^6$$

Credit: **S4 NGHS P1/2020 PRELIM Qn 6**

1.6 Exponential & Logarithms

1. The population of polar bears in the arctic is given by the formula, where t is in years

$$N = 8000\left(2 + 3e^{-\frac{t}{50}}\right)$$

Find

(a) the initial population [1]

(b) the population after 50 years [1]

(c) the least number of years it would take the population to exceed 20000 [3]

(d) the rate at which the polar bears is decreasing when $t = 10$ [2]

(e) Explain why the population of the polar bear can never fall below 16000 [2]

(f) Sketch the population-time curve in the grid below [2]

Credit: **S4 CHS P1/2021 PRELIM Qn 2**

2. (a) (i) Sketch the graph of $y = \log_2(3x+1)$ and label the point where $x = \dfrac{1}{3}$ [2]

(ii) Explain why $x > -\dfrac{1}{3}$ [1]

(b) Solve [4]

$$\log_2(3x+1) + \frac{1}{2}\log_{\sqrt{2}}(3x-1) = 1$$

Credit: **S4 MGS P2/2021 PRELIM Qn 1**

3. (a) Given that

$$\log_2 p = x \qquad \log_2 q = y$$

express the following in terms of x and y

(i) [1]

$$\log_2 1 - p + q$$

(ii) [3]

$$\log_2 \sqrt{\frac{p^5}{q^3}}$$

(iii) [4]

$$\log_{\sqrt{2}} 4p$$

(b) Solve [5]

$$4\log_4 x + 1 = 3\log_8(5-3x)$$

Credit: **S4 BVSS P1/2020 PRELIM Qn 3**

4. (a) Solve the equation [3]

$$2\log_5 x + \log_{25} 16 = \log_5(9x - 2)$$

(b) Given that [5]

$$\frac{1}{\log_a b} - \frac{1}{\log_b a} = \sqrt{293} \qquad a > b > 1$$

find the value of

$$\frac{1}{\log_{ab} a} - \frac{1}{\log_{ab} b}$$

Credit: **S4 JSS P2/2020 PRELIM Qn 4**

5. (a) The magnitude of an Earthquake is measured by the Richter scale. The magnitude, M, can be determined by the equation, where I is the intensity of seismic waves recorded by seismographs and I_0 is a constant which represents the threshold intensity

$$M = \lg\left(\frac{I}{I_0}\right)$$

(i) State the magnitude of an Earthquake such that its intensity is the same as the threshold intensity [1]

(ii) Earthquakes of magnitude 5.8 and 6.3 occured in Taiwan and Canada respectively in 2019. Find the value of the following as a power of 10, [3]

$$\frac{I_C}{I_T}$$

given that I_C and I_T represent the intensity of the Earthquakes in Canada and Taiwan respectively

(b) Solve the simultaneous equations [5]

$$2^{p-9} \div 8^q = \sqrt[4]{32^p}$$

$$\log_2 6 - \log_4(11q - 2p) = 1$$

Credit: **S4 MSS P1/2020 PRELIM Qn 5**

1.7 Trigonometry

1. Given that

$$\sin A = -\frac{4}{5} \qquad \tan B = -\frac{5}{12} \qquad \cos A > 0$$

A and B are in different quadrants. Evaluate the following, **without the use of a calculator**, the values of

(a) [1]

$$\cot A$$

(b) [2]

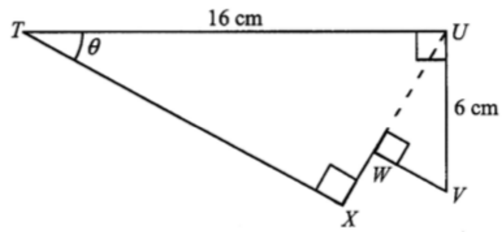
$$\cos(A+B)$$

(c) [3]

$$\sin\left(\frac{B}{2}\right)$$

Credit: **S4 MGS P1/2021 PRELIM Qn 5**

2. The diagram shows a pentagon $TUVWX$ with three fixed points, T, U and V such that $TU = 16$ cm, $UV = 6$ cm and $\angle TUV = 90°$



The lines TX and VW are perpendicular to the line UX. The angle θ can vary in such a way that the point W lies between the points U and X

(a) Show that the perimeter, P cm, of pentagon $TUVWX$ is given by [3]

$$P = 22 + 10\cos\theta + 22\sin\theta$$

(b) Express P in the form $22 + R\sin(\theta+\alpha)$, where $R > 0$ and $0° < \alpha < 90°$ [4]

(c) Explain why it is possible for the pentagon to have a perimeter of 45 cm [1]

(d) Find the values of the value of θ for which $P = 45$ [2]

Credit: **S4 SCGS P1/2021 PRELIM Qn 7**

3. (a) Show that [4]

$$\cos(A+B)\cos(A-B) = \cos^2 A + \cos^2 B - 1$$

(b) **Hence**, determine the value of $\cos 15° \cos 75°$ without the use of calculator [3]

Credit: **S4 SMSS P2/2021 PRELIM Qn 7**

4. (a) (i) Prove the identity [4]

$$\sin x \cos x + \cot x \cos^2 x = \cot x$$

(ii) **Hence**, solve, for $0 \le x \le \pi$ [3]

$$\sin 3x \cos 3x + \cot 3x \cos^2 3x = 1$$

(b) (i) On the same axes, sketch the graphs of the following, for $0 \le x \le 2\pi$ [5]

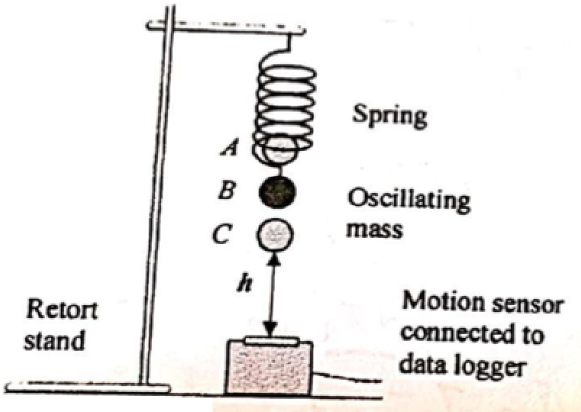
$$y = 3\sin x - 1 \qquad y = \tan\frac{x}{2}$$

(ii) Hence, state the number of solutions, for $0 \le x \le 2\pi$ [1]

$$3\sin x - 1 = \tan\frac{x}{2}$$

Credit: **S4 AHS P2/2020 PRELIM Qn 9**

5. A mass attached to a spring is pulled vertically downwards to position C from its equilibrium position B and then released. The mass will bob up to position A. Assuming that the mass never loses any energy or momentum, it will oscillate up and down between A and C



Given that C is the initial position of the mass, the distance between the mass and the motion sensor, h cm, over time, t seconds, can be modelled by the equation, where k is a constant

$$h = -3\cos(k\pi t) + 7$$

The time taken for the mass to travel from C to A is 0.25 seconds

(a) Explain why this model suggests that the distance between A and C is 6 cm [1]

(b) Show that the value of k is 4 radians per second [2]

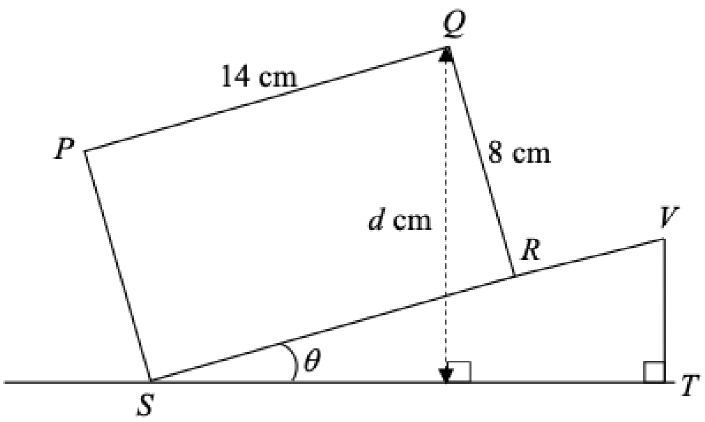
(c) Solve, for $0 \le t \le 0.5$ [3]

$$-3\cos(4\pi t) + 7 = 8$$

(d) Using your answer in part (c), find the duration when the mass is within 2 cm of A, when it travels from C to A [1]

Credit: **S4 ACS(B) P1/2020 PRELIM Qn 8**

6. The diagram shows the frontal view of a rectangular block $PQRS$, with size 14 cm by 8 cm.



The block is placed on an adjustable ramp VS such that it is tilted at an acute angle θ and $\angle VTS = 90°$. The ramp is placed on a horizontal surface ST and the perpendicular distance from Q to ST is d cm

(a) Show that [3]

$$d = 8\cos\theta + 14\sin\theta$$

(b) Express d in the form $R\sin(\theta + \alpha)$, where $R > 0$ and α is acute [2]

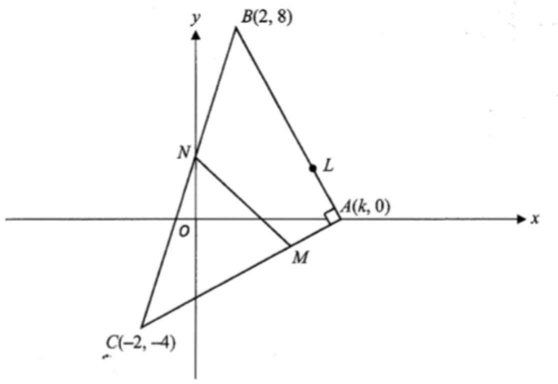
(c) The perpendicular distance from Q to ST is $\sqrt{200}$ cm. Find the smallest angle θ [3]

(d) State the maximum value of d [1]

Credit: **S4 BGSS P2/2020 PRELIM Qn 8**

1.8 Coordinate Geometry

1. **Solutions to this question by accurate drawing will NOT be accepted**



The figure shows a right-angled triangle ABC, where points A, B and C are $(k, 0)$, $(2, 8)$ and $(-2, -4)$ respectively. BC cuts the y-axis at N. M is a point on AC

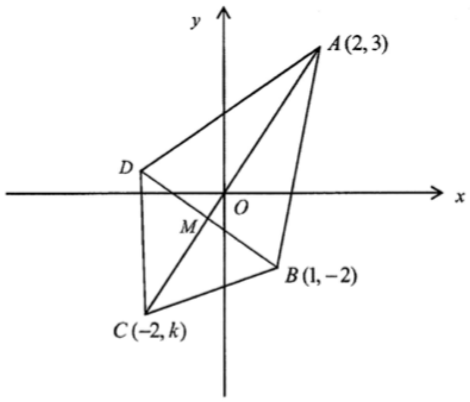
(a) Given that $k > 0$, find the value of k [3]

(b) Find the coordinates of N and show that N is the midpoint of BC [3]

(c) Given that the area of quadrilateral $ABNM$ is 25 units^2, find the coordinates of M [6]

Credit: **S4 AHS P2/2021 PRELIM Qn 6**

2. The diagram shows a kite $ABCD$ in which the line AC passes through the origin and has a gradient of 1.5



(a) Find the value of k [2]

(b) Find the equation of the line BD [3]

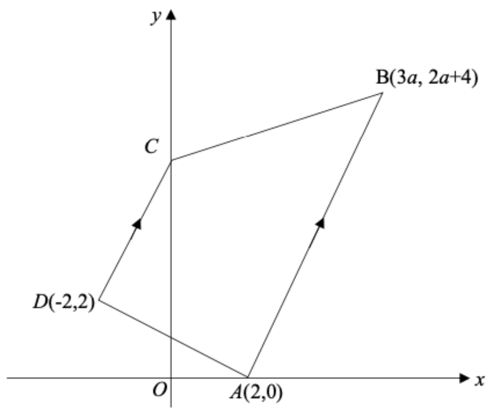
(c) The line AC and line BD meets the point M. Show that,M is $\left(-\frac{8}{13}, -\frac{12}{13}\right)$ [3]

(d) The coordinates of D are (a, b). Prove that [2]

$$13\left(a + \frac{8}{13}\right)^2 + 13\left(b + \frac{12}{13}\right)^2 = 49$$

Credit: **S4 CCHS(M) P2/2021 PRELIM Qn 7**

3. **Solutions to this question by accurate drawing will not be accepted**



The diagram shows a trapezium $ABCD$ in which AB is parallel to DC. The coordinates of the points A, B and D are $(2, 0)$, $(3a, 2a + 4)$ and $(-2, 2)$ respectively, where a is a positive integer. The length of AB is $4\sqrt{5}$ units

(a) Show that $a = 2$ [3]

(b) Find the coordinates of C [2]

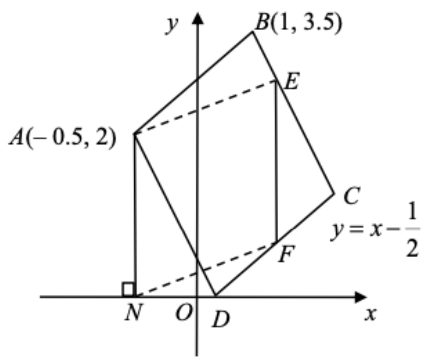
(c) Find the equation of the perpendicular bisector of AB [4]

(d) **Hence**, or otherwise, determine if C lies on the perpendicular bisector of AB [1]

(e) Find the area of the trapezium $ABCD$ [2]

Credit: **S4 CHIJ SNGS P1/2020 PRELIM Qn 10**

4. **Solutions by accurate drawing will not be accepted**



The points $A(-0.5, 2)$, $B(1, 3.5)$, C and D are the four vertices of a parallelogram. The point E lies on BC such that $3BE = BC$. The line CD has the equation $y = x - \frac{1}{2}$. Lines are drawn, parallel to the y-axis, from A to meet the x-axis at N and from E to meet CD at F

(a) Calculate the coordinates of C and E [6]

(b) Find the coordinates of F [1]

(c) Explain why $AEFN$ is a parallelogram [4]

Credit: **S4 HIHS P1/2020 PRELIM Qn 11**

1.9 Further Coordinate Geometry

1. A circle, C_1, has an equation
$$x^2 - 6x + y^2 + 10y = 66$$
   (a) Are the x-axis and y-axis tangents to C_1? Explain your answer [3]
   (b) Does the point $(2, -4)$ lie inside, on or outside of C_1? Show your working clearly [2]
   (c) A second circle, C_2, is the reflection of C_1, on the y-axis. Find the equation of C_2 [2]

   Credit: **S4 CCHS(M) P1/2021 PRELIM Qn 6**

2. The equation of a circle, centre C, is given as the following, where p and k are constants
$$x^2 + y^2 + px + \left(\frac{p}{2} + 4\right) y + k = 0$$
   It is given that C lies on the line
$$3x - 2y - 8 = 0$$
   (a) Show that $p = -4$ [2]
   (b) Find the coordinates of C [1]
   (c) Find the value of k, given that $x = -8$ is a tangent to the circle [3]
   (d) The coordinates of point A is $(14, -8)$. Show that A lies outside of the circle [2]
   (e) Point X lies on the circle such that it is furthest from A. State the geometrical relationship between points A, C and X [1]

   Credit: **S4 TKSS P2/2021 PRELIM Qn 7**

3. A circle, C_1, and another circle, C_2, pass through the same point $(0, -3)$
   (a) Given that the radius of both circles is $\sqrt{5}$ units and their centres lie on the line $y = x$, find the equation of C_1 and C_2 [3]
   (b) Circle C_1 and C_2 intersect at a point on the x-axis. Find the x-coordinate of the point of intersection of C_1 and C_2 on the x-axis [3]
   (c) Given that a point P lies on circle, C_1 and another point Q lies on circle, C_2, find the greatest distance between P and Q [3]

   Credit: **S4 BPGHS P2/2020 PRELIM Qn 10**

4. Three points are given by $P(-2, 3)$, $Q(6, 7)$ and $R(4, 11)$
   (a) Show that $\angle PQR$ is $90°$ [3]
   (b) Explain why P, Q and R lie on a circle with diameter PR [1]
   (c) Find the equation of the circle [3]
   (d) Determine whether the point $(3, 2)$ is inside or outside the circle. Justify [2]
   (e) Given that the line $3y - 4x = k$ is a normal to the circle, find the value of k [2]

   Credit: **S4 HSS P2/2020 PRELIM Qn 6**

1.10 Linear Law

1. A bacteria sample was cultured in a laboratory. The number of bacteria in the sample, y, is related to the time elasped, t minutes, by the following equation, where k and m are constants

$$y = k(2)^{\frac{t}{m}}$$

The table below shows measured values of y and t

t	10	20	30	40	50
y	2600	4300	7000	11400	19200

(a) Plot $\lg y$ against t and draw a straight line graph [3]

(b) Use your graph to estimate

(i) the value of k and of m [4]

(ii) the time taken for the number of bacteria to reach 15000 [2]

Credit: **S4 SMSS P1/2021 PRELIM Qn 10**

2. (a) The variables x and y are related in such a way that when $\frac{x}{y}$ is plotted against $\frac{1}{x}$, a straight line is obtained. The line passes through $(2, 9)$ and $(5, 3)$. Find an expression for y in terms of x [4]

(b) The table shows experimental values of two variables, x and y [6]

x	2	4	6	8
y	8.48	5.99	4.90	4.24

It is known that x and y are related by the equation $x^n y = k$, where n and k are constants. Draw a suitable straight line graph to represent the above data and use it to estimate the values of n and k

Credit: **S4 AHS P1/2020 PRELIM Qn 11**

3. The population, P, of a small town decreases with time, t years. It is known that P and t are related by the equation, where P_0 and k are constants

$$P = P_0 e^{-kt}$$

The table below shows measured values of P and t

t	6	9	12	15	18
P	274	203	151	112	83

(a) Plot $\ln P$ against t and draw a straight line graph [3]

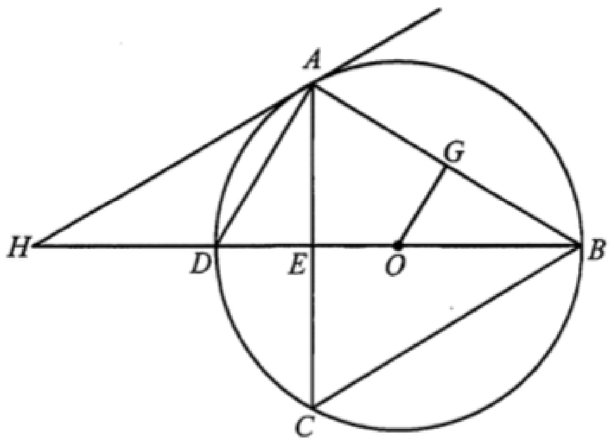
(b) Estimate the values of P_0 and k, giving your answers correct to the nearest hundred and to 1 decimal place respectively [4]

(c) Find the number of years it will take for the population of the small town to drop below 100. Give your answer correct to the nearest year [2]

Credit: **S4 HIHS P2/2020 PRELIM Qn 6**

1.11 Proofs of Plane Geometry

1. The diagram shows a circle, centre O, with points A, B, C and D lie on the circle [3]



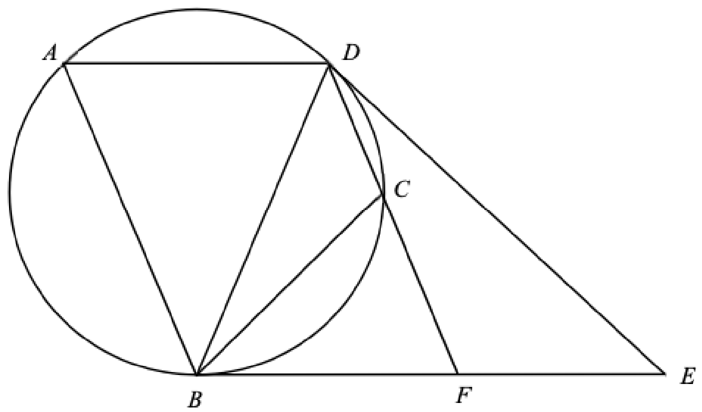
HA is a tangent to the circle. D and G are mid-points of HB and AB respectively. AD is the angle bisector of $\angle CAH$

(a) Prove that OG is perpendicular to AB [3]

(b) Prove that $\angle ABD = \angle CBD$ [3]

Credit: **S4 SCGS P2/2021 PRELIM Qn 13**

2. The diagram shows a circle passing through points A, B, C and D



The tangents from E meets the circle at B and D. Given that $AD = BF$ and $\triangle ABD$ is isosceles, where $AB = BD$. Prove that

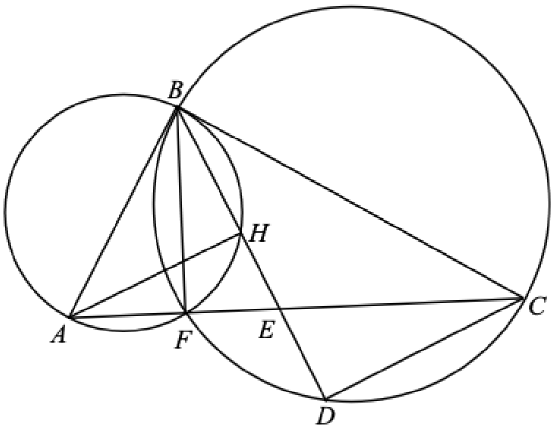
(a) $ABFD$ is a parallelogram [3]

(b) $\triangle BCE$ is similar to $\triangle DFE$ [3]

(c) $BD \times EF = CD \times DE$ [1]

Credit: **S4 AHS P1/2018 PRELIM Qn 6**

3. In the diagram, two circles intersect at B and F



BC is the diameter of the larger circle and is the tangent to the smaller circle at B. Point A lies on the smaller circle such that $AFEC$ is a straight line. Point D lies on the larger circle such that $BHED$ is a straight line. Prove that

(a) CD is parallel to AH [3]

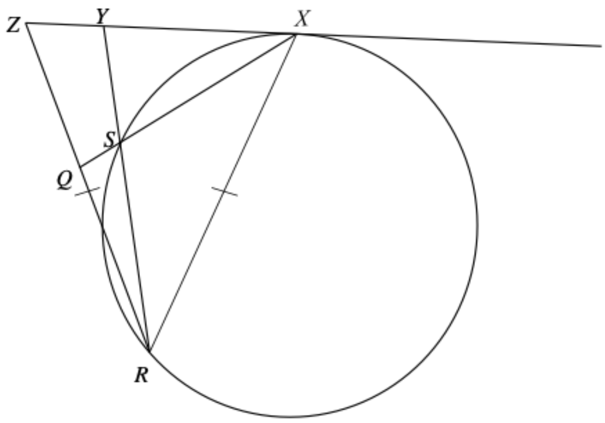
(b) AB is a diameter of the smaller circle [2]

(c) $\triangle ABC$ and $\triangle BFC$ are similar [2]

(d) $AC^2 - AB^2 = CF \times AC$ [2]

Credit: **S4 CGSS P1/2018 PRELIM Qn 12**

4. In the figure, XYZ is a straight line that is tangent to the circle at X



XQ bisects $\angle RXZ$ and cuts the circle at S. RS produced meets XZ at Y and $ZR = XR$. Prove that

(a) $SR = SX$ [3]

(b) a circle can be drawn passing through Z, Y, S and Q [4]

Credit: **S4 CHIJ SNGS P2/2018 PRELIM Qn 5**

1.12 Differentiation

1. (a) Given that [3]

$$y = \frac{e^{2x}}{\sqrt{1-4x}}$$

show that

$$\frac{dy}{dx} = \frac{4e^{2x}(1-2x)}{(1-4x)\sqrt{1-4x}}$$

(b) (i) The equation of the curve is given by for $0 \le x \le \pi$. Find $\frac{dy}{dx}$ [1]

$$y = x + \sin^2 x$$

(ii) Find the stationary point of the curve [3]

(c) Find the range of values of x such that the graph is decreasing [4]

$$y = \ln\left(\frac{x-2}{x-3}\right)^2$$

Credit: **S4 AHS P1/2021 PRELIM Qn 5, 6 & 7**

2. The equation of a curve, where $x > 0$, is

$$y = e^{3x-5x^2+\ln 2x}$$

(a) Obtain an expression for $\frac{dy}{dx}$ [2]

(b) Find the coordinates of the stationary point of the curve and leave your answer in exact form [3]

(c) Determine the nature of the stationary point of the curve [2]

Credit: **S4 NCHS P2/2021 PRELIM Qn 2**

3. (a) Given that [4]

$$y = he^x + \frac{k}{e^{2x}} \qquad \frac{d^2y}{dx^2} - 2\left(\frac{dy}{dx}\right) = e^x + 2e^{-x}$$

find the value of each of the constants h and k

(b) A cylindrical ice block of base radius r cm is melting in such a way that the total surface area, A cm^2, is decreasing at a constant rate of 72 cm^2/s. Given that the height is twice the radius and assuming that the ice block retains its shape, calculate the rate of change of r when $r = 5$ [4]

Credit: **S4 ANDSS P2/2020 PRELIM Qn 3 & 4**

4. An open water tank has a rectangular base of length $2x$ cm and breadth x cm. The height of the water tank is h cm. It is given that the total surface area of the tank is 2700 cm^2

   (a) Express the height of the tank, h cm, in terms of x [2]

   (b) Show that the volume of the tank, V cm^3 is given by [1]

$$V = 900x - \frac{2}{3}x^3$$

   (c) Given that V varies with x, find the value of x for which V is maximum [4]

   (d) Find the maximum value of V [1]

Credit: **S4 JSS P1/2020 PRELIM Qn 8**

5. (a) Given that

$$y = 3xe^{-2x}$$

   find

   (i) [3]

$$\frac{dy}{dx}$$

   (ii) the value of p if [4]

$$p = e^{2x}\left(\frac{d^2y}{dx^2} + \frac{dy}{dx} - 2y\right)$$

   (b) A curve has the equation

$$y = \ln\left(\frac{1-\cos x}{\sin x}\right)$$

   (i) Show that [5]

$$\frac{dy}{dx} = \csc x$$

   (ii) A point P moves along the curve such that $0 < x < \frac{\pi}{2}$. Find the exact value(s) of x when the rate of increase of y is twice the rate of increase of x [3]

Credit: **S4 NHHS P2/2020 PRELIM Qn 8**

1.13 Integration

1. The function f is defined for $x \neq -\dfrac{1}{3}$ and is such that [7]

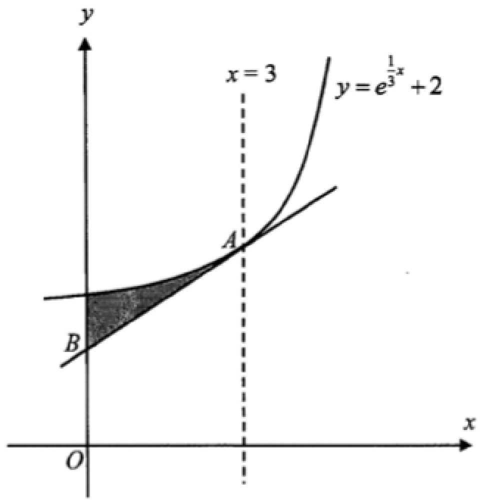
$$f''(x) = 4e^{2x} + \frac{9}{(3x+1)^2}$$

Given that $f'(0) = -1$ and $f(0) = 2$, find an expression for $f(x)$

Credit: **S4 CHS P2/2021 PRELIM Qn 7**

2. The diagram shows part of the curve

$$y = e^{\frac{1}{3}x} + 2$$



The tangent to the curve at A intersects the y-axis at B

(a) Find the exact area of the shaded region bounded by the tangent AB, the curve and the y-axis [8]

(b) Find the equation of the normal to the curve at $x = 0$ [3]

Credit: **S4 CHIJ SNGS P2/2021 PRELIM Qn 11**

3. Given that [4]

$$y = A - B\cos 4x - \frac{1}{2}\sin 2x \qquad \frac{d^2y}{dx^2} + 4y = 3\cos 4x + 1$$

find the value of each of the following constants A and B

Credit: **S4 TKSS P2/2021 PRELIM Qn 3**

4. It is given that

$$\int_0^2 f(x)\ dx = 4 \qquad \int_2^5 f(x)\ dx = 12$$

(a) Evaluate [1]

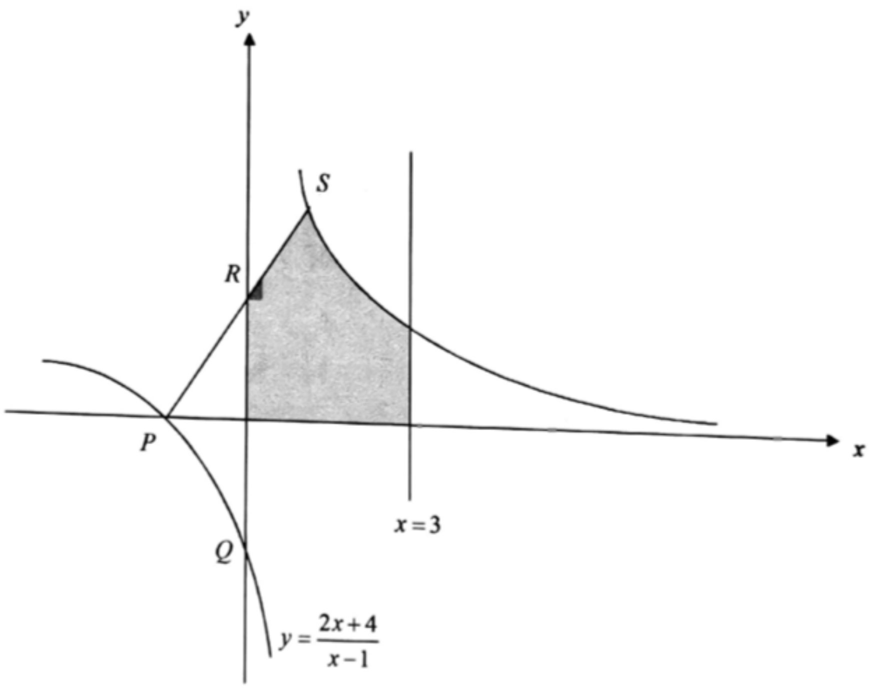
$$\int_0^5 f(x)\ dx$$

(b) Find the value of m for which [3]

$$\int_0^2 \left[f(x) + mx^2\right]\ dx = \int_5^2 f(x)\ dx$$

Credit: **S4 BGSS P1/2020 PRELIM Qn 3**

5. The diagram below shows the curve $y = \dfrac{2x+4}{x-1}$ which cuts the x-axis at P, the y-axis at Q.



The normal to the curve at P meets the y-axis at R. S is the point where the normal meets the curve again

(a) Find the coordinates of P and of Q [2]

(b) Find the coordinates of R and of S [7]

(c) Find the area of the shaded region [5]

Credit: **S4 BVSS P1/2020 PRELIM Qn 10**

1.14 Differentiation & Integration

1. (a) Show that [3]

$$\frac{d}{dx}\left(\tan^3 x\right) = 3\sec^4 x - 3\sec^2 x$$

   (b) Hence, evaluate [5]

$$\int_0^{\frac{\pi}{4}} \sec^4 x - 2\sec^2 x \; dx$$

   Credit: **S4 ANDSS P2/2021 PRELIM Qn 3**

2. (a) Express the following in partial fractions [5]

$$\frac{2x^3 - 20x^2 - 17x - 10}{(x^2-4)(2x^2+1)}$$

   (b) Differentiate the following with respect to x [2]

$$\ln\left(2x^2+1\right)$$

   (c) **Hence**, find [3]

$$\int \frac{2x^3 - 20x^2 - 17x - 10}{(x^2-4)(2x^2+1)} \; dx$$

   Credit: **S4 ANDSS P2/2020 PRELIM Qn 6**

3. (a) Show that [3]

$$\frac{d}{dx}\left[(x+3)\sqrt{2x-3}\right] = \frac{kx}{\sqrt{2x-3}}$$

   where k is a constant to be found

   (b) Hence, find [2]

$$\int \frac{x}{\sqrt{2x-3}} \; dx$$

   Credit: **S4 MFSS P1/2020 PRELIM Qn 11**

4. A curve, $y = f(x)$ is such that

$$f''(x) = 24\sin 4x - 12\cos 2x$$

   This curve has a stationary point $\left(\frac{\pi}{4}, 1\right)$. Show that, where k, p and q are constants to be determined,

$$f''(x) + 4f(x) = k\sin px + q$$

   Credit: **S4 MGS P1/2020 PRELIM Qn 12**

1.15 Kinematics

1. The velocity, v m/s, of a particle moving in a straight line, t seconds after passing through a fixed point, O, is given by
$$v = \frac{27}{2(3t+1)^2} - \frac{3t+1}{2}$$
   (a) Find the initial acceleration of the particle [2]
   (b) Determine, with appropriate working, whether the velocity of the particle is increasing or decreasing [2]
   (c) Find the average speed of the particle during the first 6 seconds [6]

   Credit: **S4 NCHS P1/2021 PRELIM Qn 12**

2. A particle P moves in a straight line, so that t seconds after leaving a fixed point O, its velocity, v m/s, is given by
$$v = 10e^{-2t} - 3$$
   (a) Find the initial velocity of P [1]
   (b) Find the acceleration of P when $t = 1$ [2]
   (c) Find the value of t when P is at instantaneous rest [3]
   (d) Find the total distance travelled by P before it comes to instantaneous rest [4]
   (e) Explain why the value of v is always greater than -3 [1]

   Credit: **S4 CHIJ SNGS P2/2020 PRELIM Qn 11**

3. A particle moves in a straight line, so that, t seconds after passing a fixed point O, its velocity, v m/s is given by
$$v = 2e^{0.1t} - 6e^{0.1-0.4t}$$
   The particle comes to an instantaneous rest at the point A
   (a) Show that the particle reaches A when $t = 2\ln 3 + \frac{1}{5}$ [3]
   (b) Find the acceleration of the particle at A [3]
   (c) Find the distance OA [4]
   (d) Explain whether the particle is again at O at some instant during the sixth second after first passing through O [2]

   Credit: **S4 GESS P1/2020 PRELIM Qn 7**

4. A particle moves in a straight line so that its velocity, v m/s, is given by, where t is the time in seconds after the start of motion
$$v = 2t^2 - 8t + 6$$
   At $t = 2$, the displacement of the particle from a fixed point O is 1 m. Find
   (a) the times when the particle is instantaneously at rest [2]
   (b) the minimum velocity of the particle and explain the significance of the answer obtained [3]
   (c) the average speed travelled by the particle in the first 5 seconds [4]

   Credit: **S4 MGS P2/2020 PRELIM Qn 9**

END OF PRACTICE QUESTIONS

2 Final Answers

2.1 Quadratic Equations & Inequalities

1. (a) Explain
   (b) $2-2\sqrt{6}<a<2+2\sqrt{6}$
2. (a) $p>2$
   (b) Shown
3. (a) $\left(x-\frac{1}{2}\right)^2+\frac{3}{4}$
   (b) Shown
4. (a) $x<-5$ and $x>\frac{1}{3}$
   (b) $c>-2$

2.2 (Indices) & Surds

1. (a) $\frac{5}{8}$
   (b) $\frac{5-\sqrt{15}}{4}$
2. $\left(2\sqrt{5}-1\right)$ cm
3. $\left(21-9\sqrt{3}\right)$ cm
4. $a=\frac{9}{19}$ and $b=-\frac{4}{19}$

2.3 Polynomials

1. (a) $f(x)=(x+1)(3x-4)(3x-1)$
   (b) Graph
   (c) $-1\leq x\leq\frac{1}{3}$ and $x\geq\frac{4}{3}$
2. (a) 300
   (b) $m=4$ or $m=25$
3. (a) 3 solutions
   (b) $f(x)=3x^4-12x^3-12x^2+57x+18$
   (c) $-11\frac{13}{16}$
4. (a) $x=1$ or $x=1.69$ or $x=-1.19$
   (b) 256
5. (a) $p=-3$ and $q=4$
   (b) $-26\frac{1}{4}$
   (c) 2 solutions

2.4 Partial Fractions

1. (a) $4+\frac{2}{x}-\frac{1}{x^2}-\frac{1}{x+1}$
   (b) $4x+2\ln x+\frac{1}{x}-\ln(x+1)+c$
2. (a) $\frac{2}{x}+\frac{3}{x^2}+\frac{1}{2x-1}$
   (b) Shown
3. (a) Shown
   (b) $-\frac{3}{5(2x-1)}+\frac{14}{5(x-3)}-\frac{7}{(x-3)^2}$
   (c) $-\frac{3}{5}\ln(2x-1)+\frac{28}{5}\ln(x-3)+\frac{14}{x-3}+c$
4. (a) $(x+2)\left(x^2-2x+4\right)$
   (b) $3+\frac{x^2+14x-12}{x^3+8}$
   (c) $h=3-\frac{3}{x-2}+\frac{4x}{x^2-2x+4}$

2.5 Binomial Theorem

1. (a) $x^{5n}+2nx^{5n-11}+2n(n-1)x^{5n-22}+...$
   (b) Shown
   (c) Shown
2. (a) $70a^4$
   (b) $210a^4-\frac{112}{a^2}$
3. (a) $1+7x+21x^2+35x^3+...$
   (b) $\binom{9}{r}(-2)^r x^{18-5r}$
   (c) $18-5r$
   (d) -637
4. (a) $p=\frac{1}{2}$
   (b) 512

2.6 Exponential & Logarithms

1. (a) 40000
   (b) 24800
   (c) 90 years
   (d) 393 polarbears/year
   (e) Explain
   (f) Graph
2. (a) (i) Graph
   (ii) Explain
   (b) $x = \frac{1}{\sqrt{3}}$
3. (a) (i) $2^y - 2^x$
   (ii) $\frac{1}{2}(5x - 3y)$
   (iii) $2(2 + x)$
   (b) $x = 1$
4. (a) $x = \frac{1}{4}$ or $x = 2$
   (b) $-\sqrt{293}$
5. (a) (i) 0
   (ii) $10^{0.5}$
   (b) $p = -\frac{72}{5}$ and $q = -\frac{9}{5}$

2.7 Trigonometry

1. (a) $-\frac{3}{4}$

   (b) $-\frac{16}{65}$

   (c) $\frac{5\sqrt{26}}{26}$

2. (a) Shown

   (b) $P = 22 + 2\sqrt{146}\sin(\theta + 24.4^\circ)$

   (c) Explain

   (d) $\theta = 47.7^\circ$ and $\theta = 83.4^\circ$

3. (a) Shown

   (b) $\frac{1}{4}$

4. (a) (i) Shown

   (ii) $x = \frac{\pi}{12}$ $x = \frac{5\pi}{12}$ $x = \frac{3\pi}{4}$

   (b) (i) Graph

   (ii) 3 solutions

5. (a) Shown

   (b) Shown

   (c) $t = 0.152$ s and $t = 0.348$ s

   (d) 0.0980 s

6. (a) Shown

   (b) $d = \sqrt{260}\sin(\theta + 29.7^\circ)$

   (c) 31.5°

   (d) $d_{\max} = 2\sqrt{65}$

2.8 Coordinate Geometry

1. (a) $k = 6$
   (b) Shown
   (c) $M(4, -1)$
2. (a) $k = -3$
   (b) $y = -\frac{2}{3}x - \frac{4}{3}$
   (c) $M\left(-\frac{8}{13}, -\frac{12}{13}\right)$
   (d) Shown
3. (a) $a = 2$
   (b) $C(0, 6)$
   (c) $y = -\frac{1}{2}x + 6$
   (d) Yes
   (e) 30 units^2
4. (a) $C\left(2, 1\frac{1}{2}\right)$ $E\left(1\frac{1}{3}, 2\frac{5}{6}\right)$
   (b) $F\left(1\frac{1}{3}, \frac{5}{6}\right)$
   (c) Shown

2.9 Further Coordinate Geometry

1. (a) Not tangents
   (b) Inside
   (c) $(x+3)^2 + (y+5)^2 = 100$
2. (a) Shown
   (b) $C(2, -1)$
   (c) $k = -95$
   (d) Outside
   (e) ACX is a straight line
3. (a) $(x+1)^2 + (y+1)^2 = 5$ and $(x+2)^2 + (y+2)^2 = 5$
   (b) $x = -3$
   (c) 5.89 units
4. (a) Shown
   (b) Explain
   (c) $(x-1)^2 + (y-7)^2 = 25$
   (d) Outside
   (e) $k = 17$

2.10 Linear Law

Note:
Questions 1(b), 2(b) & 3(b),(c) are suggested answers, range is ± 0.2

1. (a) Graph
   (b) (i) $k = 1580$ $m = 14.3$
   (ii) $t = 46.6$ minutes
2. (a) $y = \dfrac{x^2}{13x - 2}$
   (b) $k = 12.0$ $n = \dfrac{1}{2}$
3. (a) Graph
   (b) $P_0 = 500$ $k = \dfrac{1}{10}$
   (c) 17 years

2.11 Proofs in Plane Geometry

1. Prove
2. Prove
3. Prove
4. Prove

2.12 Differentiation

1. (a) Shown
   (b) (i) $\left(\dfrac{3\pi}{4}, \dfrac{3\pi + 2}{4}\right)$
   (ii) $x < 2$ and $x > 3$
   (c)
2. (a) $2e^{3x-5x^2}\left(-10x^2 + 3x + 1\right)$
   (b) $\left(\dfrac{1}{2}, e^{\frac{1}{4}}\right)$
   (c) Maximum
3. (a) $h = -1$ $k = \dfrac{1}{4}$
   (b) $\dfrac{6}{5\pi}$ cm/s
4. (a) $h = \dfrac{1350 - x^2}{3x}$
   (b) Shown
   (c) $15\sqrt{2}$
   (d) $9000\sqrt{2}$
5. (a) (i) $3e^{-2x}(-2x + 1)$
   (ii) -9
   (b) (i) Shown
   (ii) $\dfrac{\pi}{6}$ rad

2.13 Integration

1. $f(x) = e^{2x} - \ln(3x+1) + 1$
2. (a) $\left(\frac{3}{2}e - 3\right)$ units^2

   (b) $y = -3x + 3$
3. $A = \frac{1}{4}$ and $B = \frac{1}{4}$
4. (a) 16

   (b) -6
5. (a) $P(-2, 0)$ and $Q(0, -4)$

   (b) $R(0, 3)$ and $S\left(2\frac{1}{3}, 6\frac{1}{2}\right)$

   (c) 14.8 units^2

2.14 Differentiation & Integration

1. (a) Shown

   (b) $-\frac{2}{3}$
2. (a) $-\frac{3}{x-2} + \frac{2}{x+2} + \frac{4x}{2x^2+1}$

   (b) $\frac{4x}{2x^2+1}$

   (c) $-3\ln(x-2) + 2\ln(x+2) + \ln\left(2x^2+1\right) + c$
3. (a) $\frac{3x}{\sqrt{2x-3}}$

   (b) $\frac{1}{3}(x+3)\sqrt{2x-3} + c$
4. $18\sin 4x + 4$

2.15 Kinematics

1. (a) $-82\frac{1}{2}$ m/s^2
   (b) Decreasing
   (c) 5.07 m/s
2. (a) 7 m/s
   (b) -2.71 m/s^2
   (c) 0.602 s
   (d) 1.69 m
   (e) Shown
3. (a) Shown
   (b) 1.27 m/s^2
   (c) 4.81 m
   (d) Shown
4. (a) $t = 1$ or $t = 3$
   (b) -2 m/s Opposite direction
   (c) $3\frac{11}{15}$ m/s