SECONDARY 1 MATHEMATICS
Percentage, Number Patterns, Polygons
September EOY Revision

Name: _____   Date: _____

Percentage

- A way of expressing a number as a fraction of 100

- Uses the percent sign "%", for example, 50% means 50 out of 100

- Mathematically, to convert a fraction or decimal into percentage, we multiply the fraction or decimal by 100

- **Note!** If a bag costs \$2, it is **incorrect to write** $50\% = \$1$, since mathematically, $50\% = 0.5$

- Percentage change $= \dfrac{\text{New value} - \text{Old value}}{\text{Old value}} \times 100$

Worked Example 1

Paul bought a total of 60 fruits for making fruit tarts. Out of all the pens bought, 8 of them were peaches, 19 of them were pears and the remaining were apples.

What percentage of the fruits are apples?

Worked Example 2

Jason had $\$275$ left after he bought a new toy which cost 60% of his money. He then spent 20% of his remaining money on new clothes.

(a)   How much money did he have at first?

(b)   How much money is he left with now?

<u>Worked Example 3</u>

A lecture theatre has 95 students and 51 of them are girls.

(a)   Find the percentage of boys in the lecture theatre.

9 girls walked out of the lecture theatre.

(b)   Find the percentage change in the number of boys in the lecture theatre.

It is given that k more girls need to leave the lecture theatre such that the number of boys is at least double the number of girls.

(c)   Find the least value of k.

[Solution]

(a)   Number of boys $= 95 - 51 = 44$

Percentage of boys $= \frac{44}{95} \times 100 = 46.3\%$ (3sf)

(b)   Total students $= 95 - 9 = 86$

Percentage of boys $= \frac{44}{86} \times 100 = 51.2\%$ (3sf)

(c)   If the percentage of boys is at least double the number of girls,

percentage of boys is at least 66.6%.

Percentage of boys $= \frac{44}{86-k} = 0.66667$

$44 = 57.334 - 0.66667k$

$k = \frac{44-57.334}{-0.66667} = 20$

Worked Example 4

A storybook goes for $45.50 per copy.

At a bookfair, it was sold at a discount of 20%.

If a customer purchases the complete book series, comprising of 3 books, a further 15% discount would be given on the total cost.

(a)   Calculate the discounted price of 1 storybook.

(b)   Peter bought 4 storybooks. How much would he pay for the books?

Worked Example 5

(a)   The selling price of a sofa is 2502 after a 9% GST.
        Calculate the price of the sofa before GST.

(b)   Thomas sold a watch to Melissa at a profit of 20% and Melissa sold it to John at a profit of 30%. If John paid 920 for the watch, how much did Thomas pay for the watch?

Practice on your own! ☺

1. Express the following fractions / decimals as a percentage.

   (i) 0.35   (ii) 1.02   (iii) $\frac{0.03}{10}$

2. Express the following percentages as a decimal

   (i) 0.04%   (ii) $\frac{54}{60}$%   (iii) 52.1%

3. If 13% of the number is 1157, find the original number.

4. If $\frac{11}{4}$ of a number is 220 what is 36% of the number?

5. A shopkeeper bought 50 shirts for $650, and initially sells them at $29.90 each. If he gives a 20% discount per shirt, calculate his decrease in profit.

6. A couple bought a TV for $2800 before GST. They were charged 9% GST, paid a 10% downpayment and the remaining amount in monthly installments over a period of 5 years. Calculate the amount paid by the couple per month.

Number Sequences / Patterns

- A number sequence is an ordered list of numbers, generated according to a number pattern.

- Each number in a sequence is called a term.

- A general term of a sequence is the n-th term of the sequence. This can generate any term in the sequence, by substituting the corresponding value of n.

- When given a pattern, take a while to try and understand the problem, before generalizing the result and finding the general term

- Check with the next few terms to see if your formula was correct!

Worked Example 1

The figure below shows the first three patterns in a sequence. Each of the patterns is made up of similar squares.

Pattern 1   Pattern 2   Pattern 3
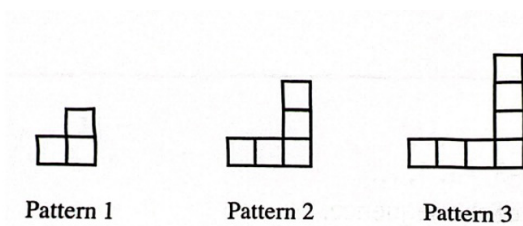
(a)   Derive a relationship between

(i)   the number of rows and the pattern number

(ii)   the number of rows and the total number of squares

(iii) the pattern number and the perimeter of the figure

(b)   Hence, find the number of rows, total number of squares, and perimeter of

(i)   pattern 10

(ii)   pattern 142

Worked Example 2

Lucas makes a pattern of sequence using some beads.

Diagram 1   Diagram 2   Diagram 3   Diagram 4

(a)   Draw diagram 5 in the space below.

(b)   Find the number of dots in diagram 9.

(c)   Find an expression, in terms of n, for the number of dots in diagram n.

(d)   Explain why it is not possible to have a diagram with 128 dots.

<u>Worked Example 3</u>

Let T_n be the nth term in the number sequence $1, 4, 7, 10, 13...$

(i)   Find an expression for T_n.

Let U_n be the nth term in the number sequence $\frac{1}{2}, 1, \frac{7}{6}, \frac{5}{4}, \frac{13}{10} ...$

(ii)   By considering your answer in part (i), or otherwise, find an expression for U_n.

A student observes that Q_n is an increasing sequence, and claims there are two numbers, Q_k and Q_{2k}, such that the difference between the two numbers is $\frac{1}{44}$.

(iii)   Form an equation in k and solve for this value of k.

[2022/TJC/EOY]

(i) $T_n = 1 + 3(n-1) = 3n - 2$

(ii) U_n can also be written as $\frac{1}{2}, \frac{4}{4}, \frac{7}{6}, \frac{10}{8}, \frac{13}{10}, ...$

        The numerator can be written as T_n

        The denominator can be written as $2n$

        Hence, $U_n = \frac{3n-2}{2n}$

(iii) $U_k = \frac{3k-2}{2k}$

        $U_{2k} = \frac{6k-2}{4k}$

        $U_{2k} - U_k = \frac{1}{44}$

        $\frac{6k-2}{4k} - \frac{3k-2}{2k} = \frac{1}{44}$

        $\frac{6k-2}{4k} - \frac{6k-4}{4k} = \frac{1}{44}$

        $44(6k - 2 - 6k + 4) = 4k$

        $44(2) = 4k$

        $k = 22$

        We can check our answers using U_{22} and U_{44}

        $U_{22} = \frac{66-2}{44} = \frac{64}{44}$

 $U_{44} = \frac{132-2}{88} = \frac{130}{88} = \frac{65}{44}$

        $U_{44} - U_{22} = \frac{65}{44} - \frac{64}{44} = \frac{1}{44}$
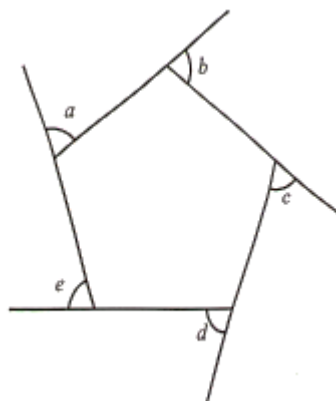
Polygons

- A polygon is a closed plane figure formed by 3 or more line segments

- Convex polygon: each of its interior angles is less than 180°

- Concave polygon: some of its interior angles is more than 180°

- Regular polygon: all the sides are of equal length

- A polygon is named according to the number of sides it has

Number of sides	Name of Polygon
3	Triangle
4	Quadrilateral
5	Pentagon
6	Hexagon
7	Heptagon
8	Octagon
9	Nonagon
10	Decagon

- Sum of interior angles in n-gon $= (n - 2) \times 180^o$

- One interior angle in n-gon $= \frac{(n-2) \times 180^o}{n}$
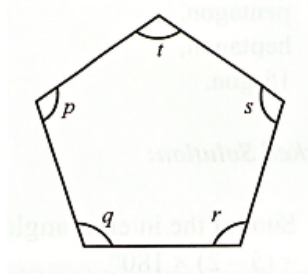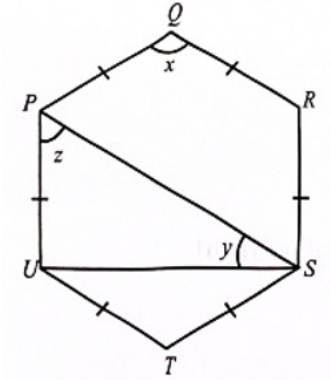

- Sum of all exterior angles $= 360^o$

- One exterior angle of n-gon $= \frac{360^o}{n}$

Worked Example 1

In the figure below, $PQRSTU$ is a regular polygon. Find the sum of all unknown angles.

Worked Example 2

Alan claims that 3 of the exterior angles of an irregular nonagon are $(x + 30)°$, $(2x - 6)°$ and $(5x + 14)°$, and the remaining exterior angles are $55°$ each. Is his claim accurate? Justify your answer.

Worked Example 3

In the figure shown below, $PQRSTUVW$ is a regular octagon, $PWXY$ is a squre, and $XWYZ$ is part of a regular n-gon.
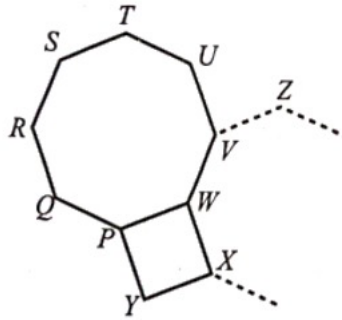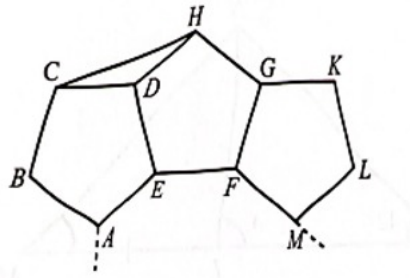

Find,

(i) $\angle VWX$

(ii) $\angle UVZ$

(iii) $\angle VPX$

(iv)   the value of n

Worked Example 4

In the figure shown below, $ABCDE$, $DEFGH$ and $FGKLM$ are 3 similar regular pentagons.

$AEFM$ is part of a n-sided regular polygon.

Find,

(i) $\angle AED$,

(ii) $\angle DCH$,

(iii)   the value of n.