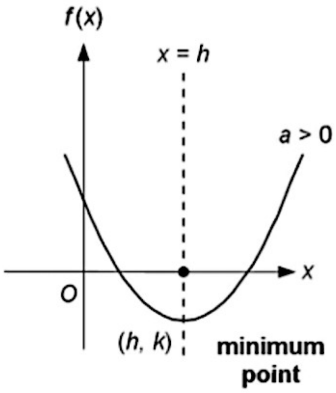
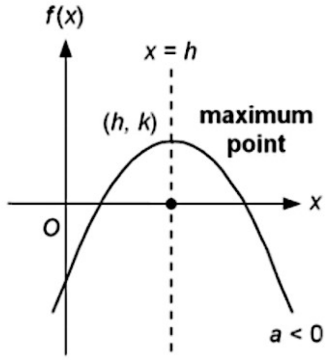
SECONDARY 3 MATHEMATICS
Quadratic Graphs, Similarity of 3D Figures, Circular Measure
September EOY Revision

Quadratic Graphs

1. Sketching of graphs $y = ax^2 + bx + c$

   - Identify the shape of the quadratic graph based on the signature of the quadratic coefficient α.
   - y- intercepts of the graph can be obtained by solving for $x = 0$.
   - x- intercepts of the graph can be obtained by solving for $f(x) = 0$.
   - Determine the Maximum/Minimum values with the above mentioned 3 methods.





Curve	Assumptions	Feature
$y = \pm(x-h)^2 + k$	$h\,,k > 0$	(h,k) is the turning point of the graph
$y = \pm(x-p)(x-q)$	$p\,,q > 0$	$(p,0)$ and $(q,0)$ are the x-intercepts

2. Find the roots of quadratic equations

a) Factorisation

- This method works when the equation is easily factorized.
- Factorise $f(x) = ax^2 + bx + c = 0$ to get the product of its linear factors $a(x - p)(x - q) = 0$ where p and q are known as the roots of the equation.
- Use the multiplication property that when $ab = 0 \Rightarrow a = 0 \text{ or } b = 0$

b) Quadratic Formula

- The roots of the equation $y = ax^2 + bx + c = 0$ can be obtained with

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}, a \neq 0$$

c) Completing the Square

- Rearrange the quadratic equation $f(x) = ax^2 + bx + c = 0$ in the form of

$$f(x) = a(x - h)^2 + k = 0$$

- Make $(x - h)^2$ the subject of the equation

$$f(x) = (x - h)^2 = \frac{k}{a}$$

Take note that when square rooting on both sides, must take both the positive and negative of a square root

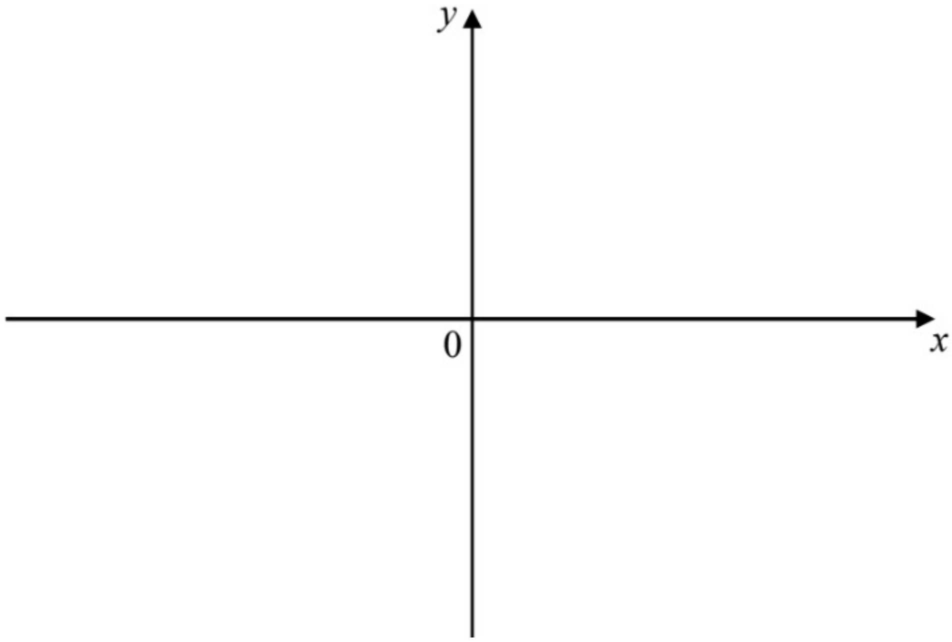
- Take the square roots of both sides of the equation and solve for the values of x

$$x = h + \sqrt{\frac{k}{a}} \qquad x = h - \sqrt{\frac{k}{a}}$$

1 (a) Express $x^2 + 16x - 30$ in the form $(x + h)^2 - k$, where h and k are constants to be determined.

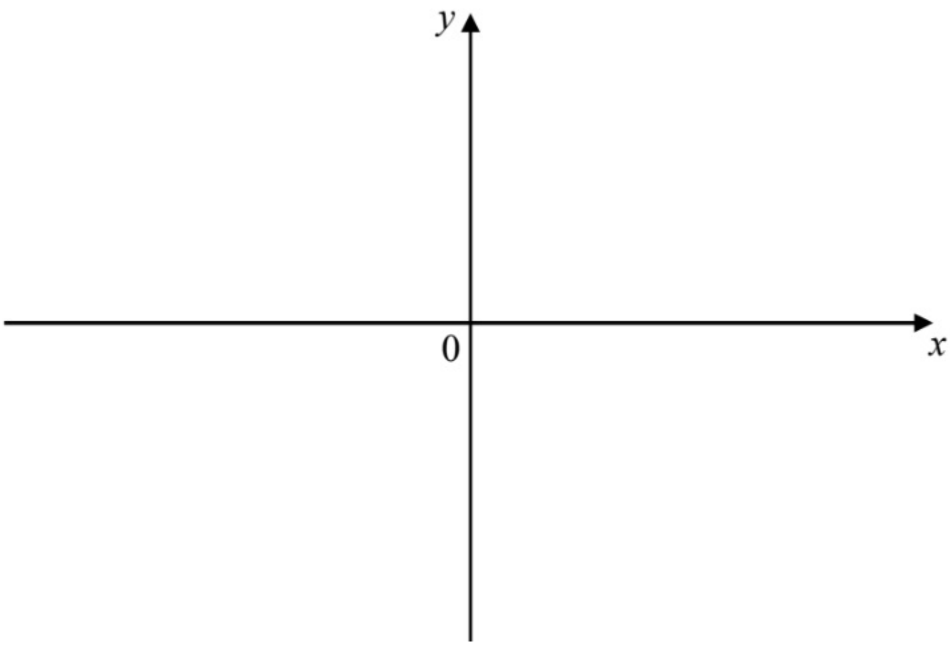
(b) Hence, solve the equation $x^2 + 16x - 30 = 0$, giving your answers correct to 2 decimal places.

(c) Sketch the graph oh $y = x^2 + 16x - 30$, showing the turning point and y-intercept clearly.



2 (a) Express $-x^2 - 6x - 10$ in the form of $a(x-p)^2 + q$, where a, p and q are integers to be determined.

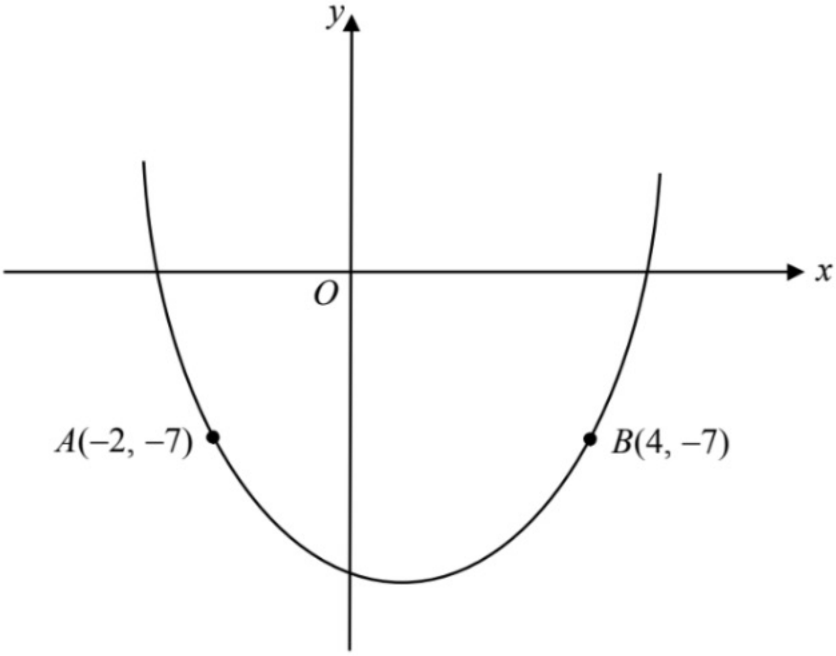
(b) Sketch the graph of $y = -x^2 - 6x - 10$ on the axes below. Indicate clearly the coordinates of the point where the graph crosses the y-axis and the turning point.



(c) Hence, explain without solving, why there are no solutions of x for the equation $-x^2 - 6x = 10$

3 The figure shows the graph of $y = (x - p)^2 + q$.

Points $A(2,7)$, $B(4,7)$ lie on the curve.



(a) Find the values of p and of q.

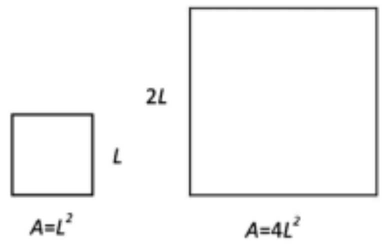
(b) Hence, find the coordinates of the x-intercepts of the curve.

Similarity of 3D Figures

Remember – If the figures are similar, we do not need to find the dimensions of the figure, nor use area/volume formulas to find the area and volume!

Area of similar planar figures

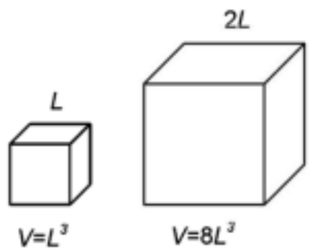
If 2 figures are similar, then the ratio of their areas is equal to the square of the ratio of the lengths of any pairs of corresponding sides.



$$\frac{A_1}{A_2} = \left(\frac{l_1}{l_2}\right)^2 = \frac{(l_1)^2}{(l_2)^2}$$

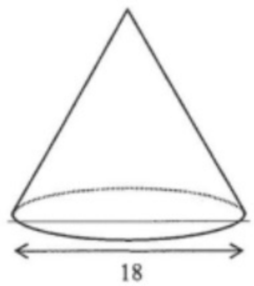
Volume of similar planar figures

If 2 solids are similar, then the ratio of their volumes is equal to the cube of the ratio of the lengths of any pairs of corresponding sides



$$\frac{V_1}{V_2} = \left(\frac{l_1}{l_2}\right)^3 = \frac{(l_1)^3}{(l_2)^3}$$

4 The figure below shows a piece of wood that is in the form of a right circular cone with base diameter of 18 cm. The curved surface area of the cone is 135π cm^2.

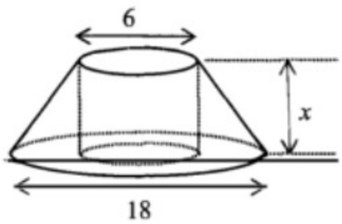


(a) Find the height of the cone

The cone is cut into two portions with a horizontal cut. The upper portion is a cone of base diameter 6 cm and the lower portion is a frustum of height x cm.

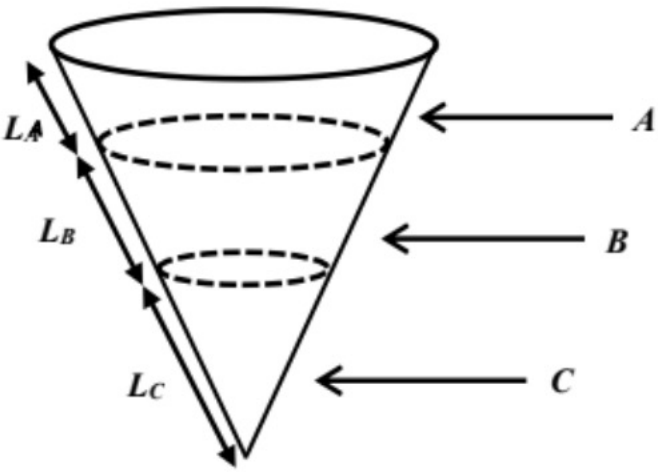
(b) Find the height of the frustum.

A right cylindrical hole of diameter 6 cm is drilled through the frustum as shown in the figure below. The wood weighs 810 kg/m^3.



(c) Find the mass, in grams, of the solid which remains in the frustum.

5 A solid cone is made up of frustum A, frustum B and cone C.
The ratio of the slant height of A: B: C is given by $2:3:5$.

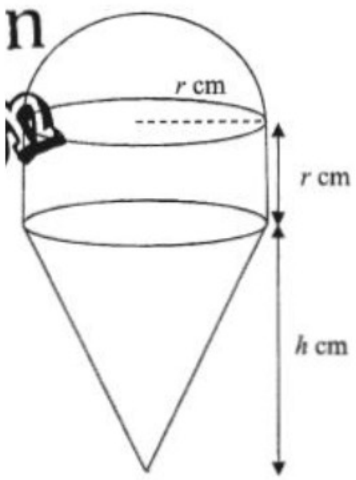


(i) Find the ratio of the curved surface area of A : curved surface area of B.

(ii) Find the ratio of volume of A : volume of whole cone

(iii) Given that the volume of A is $122\ \text{cm}^3$, calculate the volume of B.

6 The figure below shows a solid formed by joining a hemisphere of radius r cm to one end of a cylinder of height r cm. The other end of the cylinder is attached to a cone of height h cm.



Solid *A*

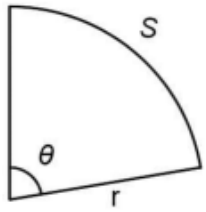
(a) Find, in terms of π and r, the total volume of the hemisphere and the cylinder.

(b) The volume of the cone is half the volume of the entire solid. Show that $h = 5r$.

(c) Given that the volume of hemisphere is 54π cm^3, find the volume of the solid.

Circular Measure

The shape below is known as a sector, which is cutting a fraction out from a circle. Using fractions, we have the following formulas.



$$\text{Arc length} = \frac{\theta}{360^o} \times 2\pi r$$

$$\text{Area of sector} = \frac{\theta}{360^o} \times \pi r^2$$

Where $2\pi r$ and πr^2 gives the circumference of the circle and area of circle respectively.

Using **radians** to measure the angle instead of degrees, since $180^o = \pi$ radians, we have,

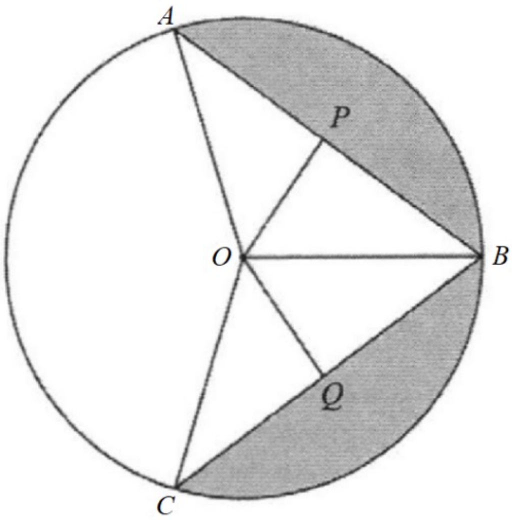
$$\text{Arc length} = \frac{\theta}{2\pi} \times 2\pi r = r\theta$$

$$\text{Area of sector} = \frac{\theta}{2\pi} \times \pi r^2 = \frac{1}{2} r^2 \theta$$

These formulas allow us to calculate the arc length and area of a sector much more easily. Remember however, that the angle has to be measured in radians instead of degrees!

7 A, B and C are points on the circle centre O and $AB = BC$.

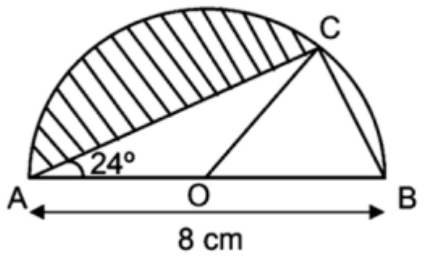
P is the midpoint of chord AB and $Q$is the midpoint of chord BC.



(a) Prove that triangle OAP is congruent to triangle OCQ.

(b) Given trat the radius of the circle is 6 cm and the obtuse angle $AOC = \frac{7\pi}{9}$, calculate the shaded area of the figure.

8 The diagram above shows a semicircle of diameter AB. It is given that $AB = 8$ cm and C is a point on the semicircle such that $\angle CAB = 24^{\circ}$.

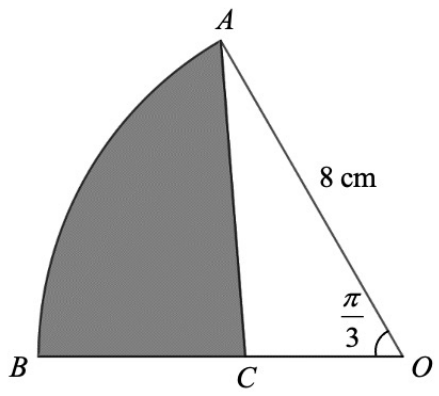


Find,

(a) the area of ΔABC,

(b) the perimeter of the shaded region subtended by the arc AC.

9 The diagram below, not drawn to scale, shows a sector *OAB* with $\angle AOB = \frac{\pi}{3}$ and radius 8 cm.



Given that point *C* lies on the line *BO*, find the possible value of the length of *CO*

(a) if the area of the shaded region is equal to the area of the non-shaded region,

(b) if the perimeter of the shaded region is equal to the perimeter of the non-shaded region.